Sample exam Game Theory & Auctions
(Web Sciences 201500025)

- Use of calculators, mobile phones, etc. is not allowed!
- This exam consists of four problems. Please start a new page for every problem.
- You have 2 h time.
- Please write name and student ID on your solutions.
- Total number of points: 36+4 = 40. Distribution of points:

1a: 3	2a: 3	3a: 3	4a: 3
1b: 3	2b: 3	3b: 3	4b: 3
1c: 3	2c: 3	3c: 3	4c: 3

Exercise 1

Decide for each of the following statements whether it is true or false. Give a short argument to justify your answer (one or two sentences, or a counterexample). *(3 points per statement)*

(a) Every two-player zero-sum game possesses a pure Nash equilibrium.

(b) In an VCG-based ad-auction, the bidder with the highest valuation can increase his payoff by not bidding truthfully.

(c) In every generalized second-price auction, bidding truthfully is a dominant strategy.

Exercise 2

Consider the game specified by the following payoff matrix and answer the following questions.

		opponent		
		ℓ	m	r
you	T	7/5	0/5	3/1
	M	8/0	1/1	4/2
	B	9/2	3/3	2/2

(a) *(3 points)* What, if any, are your dominant, and strictly dominant strategies? What, if any, are your opponent's dominant, and strictly dominant strategies?

(b) *(3 points)* Which strategies are strictly dominated by which strategies? Write down the resulting game with the strictly dominated strategies removed.

Does the resulting reduced game have strictly dominated strategies? If yes, by which strategies are they dominated? Iterate the process of removing strictly dominated strategies until no strictly dominated strategy remains.

(c) *(3 points)* Does the resulting game have pure Nash equilibria? If yes, list all pure Nash equilibria. Does the game have mixed Nash equilibria? If yes, compute and describe them.

Exercise 3

Consider the example of the following matching market.

		items		
		A	B	C
buyers	W	11	4	1
	X	8	7	6
	Y	7	5	3

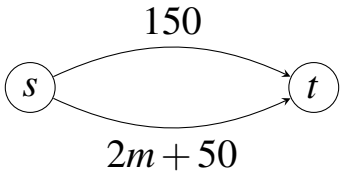
(a) *(3 points)* Run the price raising procedure to compute market-clearing prices.

(b) *(3 points)* What is the assignment of items to buyers that maximizes the total valuation of the buyers? What is its value?

(c) *(3 points)* Run the VCG procedure to compute VCG prices. (You do not have to compute all $V_{-?}^{-?}$ values, but only the three relevant ones.) Compare the resulting assignment of items to buyers with that of (b) and comment on that.

Exercise 4

Consider the following simple road network. There are 100 cars that want to go from s to t. The travel time of the upper road is always 150 minutes. The travel time of the lower road is $2m+50$ minutes if there are m cars on that road.



(a) *(3 points)* Compute the social optimum for this road network and its social cost.

(b) *(3 points)* Compute a Nash equilibrium for this road network and its social cost.

(c) *(3 points)* What can you conclude for the price of anarchy for this network routing game?