Discrete Mathematics for Computer Science, Solution/Correction standard, Sample Test, Part 1

1. (a) $\exists_{a \in A} \exists_{k \in \mathbb{Z}^+} \left[a = k^2\right]$. **[3 pt]**

   (b) $\neg \exists_{a \in A} \left[a \neq 1 \wedge \forall_{d \in \mathbb{Z}^+} \left[d \,|\, a \rightarrow (d = 1 \vee d = a)\right]\right]$. **[3 pt]**

   For each expression that is not logically equivalent to the ones above: **[0 pt]**

2. We take $\neg t$ as extra premise and prove: $\neg q$.

(1)	$\neg t$	Extra Premise
(2)	$p \rightarrow t$	Premise
(3)	$\neg t \rightarrow \neg p$	(2), L13
(4)	$\neg p$	(1),(3), R1
(5)	$\neg p \vee q$	(4), R8
(6)	$(\neg p \vee q) \rightarrow r$	Premise
(7)	r	(5),(6),R1
(8)	$\neg p \wedge r$	(4),(7),R4
(9)	$(\neg p \wedge r) \rightarrow \neg s$	Premise
(10)	$\neg s$	(8),(9),R1
(11)	$s \vee \neg q$	Premise
(12)	$\neg q$	(11),(10), R5

[6 pt]

For each forgotten Law or Rule: -1 **pt**.
If deduction contains a step that is not logically correct: at most **1 pt** for the entire exercise.

3. (a) Let $C \in \mathcal{P}(A) \cup \mathcal{P}(B)$. Then $C \in \mathcal{P}(A)$ or $C \in \mathcal{P}(B)$, so $C \subseteq A$ or $C \subseteq B$. Hence $C \subseteq A \cup B$, and so $C \in \mathcal{P}(A \cup B)$. **[3 pt]**

   (b) The statement is false.
   Counterexample: $\mathcal{U} = \{1, 2\}$, $A = \{1\}$ and $B = \{2\}$. **[1 pt]**
   Then $A \cup B = \{1, 2\}$, $\mathcal{P}(A) = \{\varnothing, \{1\}\}$ and $\mathcal{P}(B) = \{\varnothing, \{2\}\}$.
   So $\mathcal{P}(A \cup B) = \{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$ and $\mathcal{P}(A) \cup \mathcal{P}(B) = \{\varnothing, \{1\}, \{2\}\}$. **[2 pt]**