Discrete Mathematics for Computer Science
Diagnostic Test; Part 1

Duration: 60 min.
Motivate all your answers. The use of electronic devices is not allowed.
A formula sheet is included.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

1. Consider a sequence of n integers $X(n) = (x_1, x_2, \ldots, x_n)$ $(x_i \in \mathbb{Z})$.
   Give quantified expressions for the following statements.

   (a) [2 pt] The sequence $X(n)$ is decreasing.
   (an example of a decreasing sequence of 5 integers is $(8, 7, 7, 2, -4)$)

   (b) [4 pt] The greatest integer in $X(n)$ is 10.
   (e.g. the greatest integer in the 4-integer sequence $(3, 5, -1, 5)$ is 5)

2. [6 pt]
   Prove the validity of the following argument using the "Laws of Logic", the "Rules of Inference" and the supplement w.r.t. quantifiers (the N-Laws and U-Rules).

$$\frac{\begin{array}{c}\forall x\ [p(x) \to q(x)] \\ \exists x\ [p(x) \vee q(x)]\end{array}}{\therefore \exists x\ q(x)}$$

3. [6 pt]
   Let A and B be sets in a universe $\mathcal{U}$.
   Prove that: $\overline{A} \Delta \overline{B} = A \Delta B$.

Total: 18 points