UNIVERSITY OF TWENTE.

Tag : Calc1B.20-21.Exam[01-B]
Course : **Calculus 1B**
Date : Friday January 8th, 2021
Time : 13:45 – 15:45

Motivate all your open answers.

The use of electronic devices is not allowed.

The exam

There are 36 points in this exam:

Multiple Choice (8 points): Q1, Q6, and Q9.

Final Answer (8 points): Q2, Q4 and Q8.

Open Answer (20 points): Q3, Q5, Q7, Q10 and Q11.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

Question types

Multiple Choice
Only select the most appropriate answer out of the alternatives on the answer sheet.

Final answer
On the answer sheet, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer
You provide a full calculation or motivation in the text frame corresponding to the question. The calculation or motivation will be graded.

Extra writing space

If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

Continue on the next page.

1. [3 pt] Given is the function

$$f(x) = x^2 e^{-x}\,.$$

Find a formula for the Riemann sum for f on the interval $[0,3]$ by dividing $[0,3]$ into n equal subintervals and using the right-hand endpoint of each subinterval to evaluate f.

Choose from the alternatives below.

A) $\displaystyle\sum_{k=1}^{n} \frac{27k^2 e^{-\frac{3k}{n}}}{n^3}$

B) $\displaystyle\sum_{k=1}^{n} \frac{8k^2 e^{-\frac{2k}{n}}}{n^3}$

C) $\displaystyle\sum_{k=1}^{n} \frac{(k-1)^2 e^{-\frac{3k-1}{n}}}{n^3}$

D) $\displaystyle\sum_{k=1}^{n} \frac{8(k-1)^2 e^{-\frac{2(k-1)}{n}}}{n^3}$

E) $\displaystyle\sum_{k=1}^{n} \frac{8k^2 e^{-\frac{3k}{n}}}{n^3}$

F) $\displaystyle\sum_{k=1}^{n} \frac{k^2 e^{-\frac{3k}{n}}}{n^3}$

G) $\displaystyle\sum_{k=1}^{n} \frac{27(k-1)^2 e^{-\frac{3(k-1)}{n}}}{n^3}$

H) $\displaystyle\sum_{k=1}^{n} \frac{8\left(k-\frac{1}{2}\right)^2 e^{-\frac{2\left(k-\frac{1}{2}\right)}{n}}}{n^3}$

2. [2 pt] Only write your final answer in the box on the answer sheet.

Compute

$$\int_0^4 \left(1+\sqrt{16-x^2}\right)\,\mathrm{d}x$$

using the relation between area and integral.

(Hint: Sketch the graph of the integrand.)

3. [2 pt] Determine

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{2+\cos(x)} e^{t^2}\,\mathrm{d}t\,.$$

Continue on the next page.

4. [2 pt] Only write your final answer in the box on the answer sheet.

Compute

$$\int x^2 \cos(1+x^3)\,\mathrm{d}x\,.$$

5. [5 pt] Determine

$$\int_1^\infty \frac{\ln(x)}{x^2}\,\mathrm{d}x\,.$$

6. [3 pt] Compute

$$\sum_{n=0}^{\infty}\left(3\cdot\frac{2^n}{7^n}+\frac{1}{4^n}\right).$$

Choose from the alternatives below.

A) $\frac{21}{5}$

B) $\frac{63}{15}$

C) $\frac{7}{15}$

D) $\frac{20}{15}$

E) $\frac{4}{3}$

F) $\frac{4}{5}$

G) $\frac{7}{3}$

H) $\frac{83}{15}$

7. [4 pt] Determine the Taylor polynomial of order 4 generated by $f(x)=\ln(x)$ at $x=1$.

8. [4 pt] Only write your final answer in the box on the answer sheet.

Solve the following differential equation subject to the given initial condition:

$$\begin{cases} \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sin(x)}{x^2} - \dfrac{2y}{x}, & x>0 \\ y(\frac{\pi}{2}) = 0 \end{cases}$$

Continue on the next page.

9. [2 pt] Let $z = -i$ and $w = 2\,e^{i\frac{\pi}{3}}$. Calculate $z \cdot w$.

Choose from the alternatives below.

A) $-2\,e^{-i\frac{\pi}{6}}$

B) $2\,e^{i\frac{\pi}{6}}$

C) $-\sqrt{3}+i$

D) $-\sqrt{3}-i$

E) $2\,e^{-i\frac{\pi}{6}}$

F) $\sqrt{3}+i$

G) $2\,e^{-i\frac{5\pi}{6}}$

H) $-2\,e^{i\frac{\pi}{6}}$

10. [3 pt] Determine all solutions in $\mathbb{C}$ of the equation $z^5 = -32i$.

11. [6 pt] Solve the following differential equation subject to the given initial conditions:

$$y'' - y = e^{-x}\,, \quad y(0) = 0\,, \quad y'(0) = 1\,.$$

The End.

Total: 36 points