Web Science (201500025) Exam Part 1

12 December 2016, 13:45-16:45, Therm

This is a closed book exam.
Use of calculators, mobile phones, etc. is not allowed.
Please write your name and student ID on your solution pages.

Calculation of mark: 1 + 9*X/45, where X is the number of points you get for answering the questions (45 points is the maximum).

Distribution of points: Each of the questions 1, 2.1, 2.2, 3(a), 3(b), 4.1(a), 4.1(b), 4.2(a) and 4.2(b) is worth 5 points.

Question 1 (week 1)

Assume that a network satisfies the Strong Triadic Closure property. Prove that if a node in a network has at least two strong ties, then any local bridge involving this node, is a weak tie.

Question 2.1 (week 2)

Consider a Web with a link structure as depicted in the figure 1. Describe (in words) at least one variant of the PageRank algorithm which could be applied effectively to rank these pages. Argue why your variant(s) is (are) suitable for this instance of the Web.

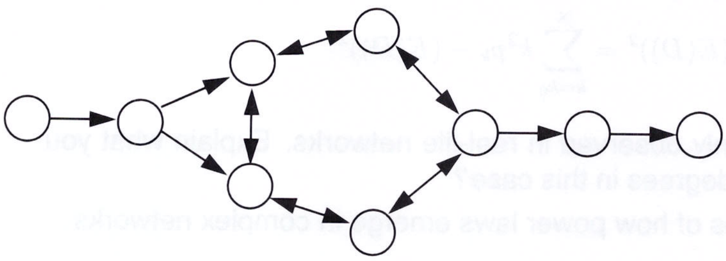

Figure 1.

Question 2.2 (week 2)

Apply the HITS page-ranking algorithm to the Web in figure 2. Compute manually the first 3 iterations of the algorithm. Can you predict what the page ranks will be, in the limit? What are the ordinal ranks (in other words, the order in which the user is shown these pages) at the end of your computation?

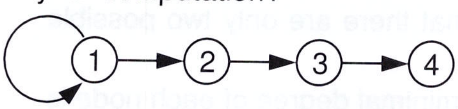

Figure 2.

Question 3 (week 3)

Consider a model in Chapter 17, where consumers occupy the interval $[0, 1]$, and their reservation price without network effects is $r(x) = 1 - \sqrt{x}$. Let p^* be the market price of the product. When proportion z of population buys the product, then the new reservation price of customer x (with network effects) is $p(x) = r(x)f(z)$, where $f(z) = \sqrt{z}$.

(a) First consider the situation without network effects. For each value $p^* > 0$ define the fraction of customers that buy the product. Explain why this point is a stable market equilibrium.

(b) Now consider the situation with network effects. For which values of p^* will we observe a tipping point? Find the expression for the tipping point z' in terms of p^* and show that this is a non-stable equilibrium.

Question 4.1 (week 4)

(a) Denote by p_k the fraction of nodes in an (undirected) network with degree k. Assume that the degree distribution obeys a power law. In particular, assume that the degree distribution is Pareto, that is, $p_k = Ck^{-\gamma-1}$, where $k \geq k_0$, $C > 0$. Show that if $\gamma \in (1, 2)$ then the expected degree

$$E(D) = \sum_{k=k_0}^{\infty} kp_k$$

is finite, while the variance of the degree

$$Var(D) = E(D^2) - (E(D))^2 = \sum_{k=k_0}^{\infty} k^2 p_k - (E(D))^2$$

is infinite. Recall that $\gamma \in (1, 2)$ is commonly observed in real-life networks. Explain what you expect to see in the empirically observed degrees in this case?

(b) Explain one of possible mechanisms of how power laws emerge in complex networks.

Question 4.2 (week 4)

Consider the model from Chapter 19 for the diffusion of a new behavior through a social network. Everyone starts with behavior B, and a threshold for switching to a new behavior A is q. Any node will switch to A if at least a fraction q of its neighbors has adopted A.

(a) A clique of size k is a graph, which contains k nodes, all connected to each other. Assume that several nodes in a clique adopted new behavior A. Describe how the process of adopting A will proceed in the clique. More precisely, prove that there are only two possible scenarios. What are the two scenarios?

(b) Give an example of a network of 10 nodes with $q = 0.2$, minimal degree of each node is 3, initially only one node adopts A, and at the end 5 nodes adopt A and 5 nodes stick to B.