Answer sheet Introduction to Mathematics

September 24, 2021, 150232

Name : Correction Model

Programme : Solutions

Student number : _____

1. *Write only the answer in the frame below:*

	V_1	V_2	V_3	V_4	
$\inf(A) =$	-1	-2	-3	-4	
$\min(A) =$	-1	-2	-3	-4	
$\max(A) =$	DNE	DNE	DNE	DNE	
$\sup(A) =$	1	1	1	1	

2. (a) *Write only the answer in the frame below:*

	V_1	V_2	V_3	V_4
$\forall x : x \in B \leftrightarrow x \notin A$	$B = \overline{A}$	$B = \overline{A}$	$A = \overline{B}$	$A = \overline{B}$

2. (b) *Write only the answer in the frame below:*

	V_1	V_2	V_3	V_4
$\forall x : x \in C \rightarrow (x \in A \wedge x \in B)$	$C \subseteq A \cup B$	$C \subseteq A \cap B$	$A \cup B \subseteq C$	$A \cap B \subseteq C$

3. *Give a full calculation/argumentation in the frame below:*

$\exists n \in \mathbb{Z}: \; a+b = 2n+1$

$\exists m \in \mathbb{Z}: \; b+c = 2m$

$(a+b)+(b+c) = 2n+1+2m$

$a+2b+c = 2n+2m+1$

$a+c = 2n+2m+1-2b = 2(n+m-b)+1,$

which is odd, since $n+m-b \in \mathbb{Z}$

4. *Give a full calculation/argumentation in the frame below:*

Let $S(n)$: $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

Basis Step

$\sum_{i=0}^{1} 2^i = 1+2 = 3$

$2^{1+1} - 1 = 4 - 1 = 3$

These are equal, so the basis step holds.

Induction Step

Suppose $S(k)$ for some $k \in \mathbb{N}$:

$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$ (IH)

We need to prove $S(k+1)$:

$\sum_{i=0}^{k+1} 2^i = 2^{k+2} - 1$

Well, $\sum_{i=0}^{k+1} 2^i = \sum_{i=0}^{k} 2^i + 2^{k+1}$

by IH → $= 2^{k+1} - 1 + 2^{k+1}$

$= 2 \cdot 2^{k+1} - 1$

$= 2^{k+2} - 1$

Therefore, the $S(n)$ is true for all $n \in \mathbb{N}$

5. (a) *Write only the answer in the frame below:*

	V_1	V_2	V_3	V_4
$\lvert A\rvert =$	2^6	2^7	2^6	2^7

5. (b) *Write only the answer in the frame below:*

V_1	V_2	V_3	V_4
$\binom{6}{3}$	$\binom{7}{3}$	$\binom{6}{2}$	$\binom{7}{2}$

5. (c) *Write only the answer in the frame below:*

V_1	V_2	V_3	V_4
$\binom{6}{3}+2^5-\binom{5}{2}$	$\binom{7}{3}+2^6-\binom{6}{2}$	$\binom{6}{2}+2^5-\binom{5}{1}$	$\binom{7}{2}+2^6-\binom{6}{1}$

Extra writing space: Give a full calculation/argumentation in the frame below: