Please note:
- ***Give your solution on a separate sheet, not as part of Test 4***
- The solution with the highest score (this extra question or the original test question) counts for 20 % of the grade of Test 2 on 22 May.
- There is a resit for the whole of Test 2 on 25 June.

3b) (Normal forms) (20 points)

Consider the relational schema $R(A,B,C,D,E)$ with functional dependencies $\mathcal{F}$, defined by

$$\mathcal{F} = \{ A \rightarrow E,\ BD \rightarrow C,\ C \rightarrow AD \}.$$

1) Which functional dependencies violate the BCNF condition? Why?

2) Apply the algorithm in the appendix to decompose R into a set of relational schemas that are all in BCNF. For each decomposition step, please give the resulting schemas with their sets of functional dependencies and their candidate keys.

3) Which of the functional dependencies in $\mathcal{F}$ were lost in the decomposition?

Appendix: BCNF decomposition algorithm

Definition of BCNF:
A relational schema is in BCNF if for every nontrivial functional dependency the left-hand side is a superkey.

Decomposition algorithm:
Let R be a relational schema with a set of functional dependencies $\mathcal{F}$.
Let $X \rightarrow Y$ be a functional dependency in $\mathcal{F}$ which violates the BCNF constraint.
- Decompose R into
    $R_1(X^+)$
    $R_2(Z)$ with $Z = \{X\}$ U {attributes of R not in X^+}.
- For i = 1,2:
    - determine $\mathcal{F}_i$ for R_i by restricting $\mathcal{F}^+$ to functional dependencies within R_i
    - if R_i is not in BCNF, recursively apply the algorithm