Exam Advanced Logic (192111092)

12 april 2016
13:45-16:45

> **Remarks:**
> - All exercises contribute equally towards the grade
> - Expected time needed: 20 minutes per exercise (averagex)
> - Allowed material: the book and the slides from the lectures. No personal notes!

Exercise 1

Consider the following propositional formula:

$$(p \to (q \to \neg q)) \to (p \to \neg q) \quad .$$

Show that this formula is valid:

1. Using a truth table.
2. Using (compositional construction of) BDDs.
3. Using the Gentzen deductive system.
4. Using the Hilbert deductive system.

Exercise 2

1. Explain precisely what it means that the Hilbert deductive system for predicate logic is complete.
2. Explain precisely what it means that validity of predicate logic formulae is undecidable, *in spite of the fact* that the Hilbert deductive system is complete.
3. Does there exist a decision procedure for the question whether an arbitrary predicate logic formula is satisfied *in all models of at most 100 elements*? Explain your answer.

Exercise 3

Suppose that you have at your disposal the vocabulary of Peano arithmetic, consisting only of the constant *zero*, the one-place function *succ* (successor), two-place functions *mult* and *add* (multiplication and addition), and the predicate *eq* (equality).

1. Define a new predicate stating that a number is odd, and write a property in predicate logic, using only the vocabulary described above, that expresses precisely when this property holds.
2. Using only the vocabulary described above (including the predicate defined in the previous item), write a property in predicate logic that expresses that every sum of two odd numbers is even.
3. Give an interpretation in which the property in the previous item is *not* satisfied, using the natural numbers as domain.

Be precise in defining your interpretations.

Exercise 4

Consider the predicates

- $male(x)$, specifying that x is a male person
- $female(x)$, specifying that x is a female person

Now consider the following predicates:

- $\forall x(male(x) \oplus female(x))$ (remembering that $\oplus$ stands for "exclusive or")
- $\exists x(male(x))$
- $\exists x(female(x))$

1. Taking Γ to be the set of predicates above, describe in words what the theory $T(\Gamma)$ consists of, and give at least one element of $T(\Gamma) \setminus \Gamma$.
2. Is $T(\Gamma)$ consistent or inconsistent? Explain your answer.
3. Taking as interpretation the domain of the biblical initial state of the world, $D = \{\mathsf{Adam}, \mathsf{Eve}\}$, is $T(\Gamma)$ complete or incomplete? Explain your answer.
4. Taking as interpretation the domain $D = \mathbb{N}$, with $male(x)$ being interpreted as "x is an even number" and $female(x)$ as "x is an odd number", is $T(\Gamma)$ complete or incomplete? Explain your answer. *(You do not have to give a formal proof.)*

Exercise 5

Consider the following assertions:

- Every pot has a lid
- A golden lid only fits on a golden pot
- The owner of a golden pot is rich
- There exists a poor pot owner

and the following conclusion:

- There exists a lid that is not golden.

1. Define predicates to encode the statements above (explaining in words what they stand for), and write the statements themselves as formulae in predicate logic.
2. Using the semantic tableau method, show that the conclusion follows from the assertions.
3. Using resolution, show that the conclusion follows from the assertions.

For the second and third exercises, include all steps required to trace your answer.

Exercise 6

Consider the 3-place Prolog predicate `remove` defined by:

```
remove(H,[],[]).
remove(H,[H|L1],L2) :- remove(H,L1,L2).
remove(H1,[H2|L1],[H2|L2]) :- remove(H1,L1,L2).
```

1. What solution(s) does this predicate provide on the query `?- remove(1,[2,1,3,1],L).`?
2. Show a step-wise calculation of the refutation that for the query `?- remove(1,[X,2,1],L).` leads to the solution `L = [2, 1]`.
3. Give a version of `remove` that is deterministic in its third parameter, such that it *only* returns the solution in which *all* occurrences of the first parameter have been removed; or explain why such a predicate cannot be specified in (pure) Prolog.