1 Geg $P(M) = 0{,}9 \quad P(C|M) = 0{,}8 \quad P(C|\bar{M}) = 0{,}1$

a. $P(\bar{M}\bar{C}) = P(\bar{C}|\bar{M})P(\bar{M}) = 0{,}9 \cdot 0{,}1 = 0{,}09$

b. $P(\bar{M}|C) = P(C|\bar{M})P(\bar{M}) \,/\, P(C)$

$$= \frac{P(C|\bar{M})P(\bar{M})}{P(C|\bar{M})P(\bar{M}) + P(C|M)P(M)} = \frac{1}{73}$$

2 a. Als we Z = X+Y noemen dan is de verdeling van Z:

z	0	1	2	3	4	som
P(Z=z)	0.15	0.10	0.60	0.10	0.05	1.00

b. De kansverdeling van X (en wegens symmetrie ook van Y) staat in onderstaande tabel:

x	0	1	2	som
P(X=x)	0.40	0.30	0.30	1.00

Dus $E(X) = \sum xP(X = x) = 1\times 0.3 + 2\times 0.3 = 0.9$.

$E(X^2) = \sum x^2 P(X = x) = 1\times 0.3 + 4\times 0.3 = 1.5$, dus $\operatorname{var}(X) = E(X^2) - (EX)^2 = 0.69$

c. $E(XY) = \sum\sum xyP(X = x \text{ en } Y = y) = 1\times 0.2 + 2\times(0.05 + 0.05) + 4\times 0.0\overset{5}{\cancel{1}} = 0.6$

$$\rho(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sigma_X\sigma_Y} = \frac{E(XY)-(EX)(EY)}{\sigma_X\sigma_Y} = \frac{0.6 - 0.9\times 0.9}{\sqrt{0.69\times 0.69}} = -\frac{21}{69} \approx -0.304$$

Er is dus een lichte negatieve correlatie tussen de waarden van X en Y.

d. $P(X = 0 \mid Y = 2) = \dfrac{P(X = 0 \text{ en } Y = 2)}{P(Y = 2)} = \dfrac{0.20}{0.30} = \frac{2}{3}$.

Evenzo $P(X = 1 \mid Y = 2) = \frac{1}{6}$ en $P(X = 2 \mid Y = 2) = \frac{1}{6}$

Dus: $E(X \mid Y = 2) = \sum xP(X = x \mid Y = 2) = \frac{1}{2}$

3 a $EX = \frac{1}{\lambda} = 4$ dus $\lambda = \frac{1}{4}$

$$P(X > 8) = \int_8^\infty f_X(x)\,dx = \int_8^\infty \tfrac{1}{4} e^{-x/4}\,dx = e^{-2} \approx 0{,}135$$

b. $E(X \mid X>4) = 4 + EX = 8$ (geheugenloosheid)

c. $E\max\{X,4\} = \int_{-\infty}^{\infty} \max\{x,4\} f_X(x)\,dx =$

$= \int_0^4 4 \cdot \frac{1}{4} e^{-x/4}\,dx + \int_4^{\infty} x \cdot \frac{1}{4} e^{-x/4}\,dx$

$= 4(1-e^{-1}) + \left(-xe^{-x/4}\Big|_4^{\infty}\right) + \int_4^{\infty} e^{-x/4}\,dx$

$= 4 + 4e^{-1}$

N.B. $E\max = E(\max \mid X<4)P(X<4) + E(\max \mid X>4)P(X>4)$
$= 4P(X<4) + 8P(X>4) = 4 + 4P(X>4)$

d. $F_Y(y) = P(Y \le y) = P(\sqrt{X} \le y) = P(X \le y^2) = F_X(y^2)$ voor $y \ge 0$

$f_Y(y) = \frac{d}{dy} F_Y(y) = 2y\, f_X(y^2)$ voor $y \ge 0$

Dus $f_Y(y) = \frac{1}{2} y\, e^{-\frac{1}{4}y^2}$, voor $y \ge 0$ (en $f_Y(y) = 0$ voor $y < 0$)

4 a $X_i \sim N(75, 100)$

$E\overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} EX_i = 75$

$\mathrm{var}\,\overline{X}_n = \frac{1}{n^2} \sum_{i=1}^{n} \mathrm{var}\,X_i = \frac{100}{n}$

$\therefore \overline{X}_n \sim N\left(75, \frac{100}{n}\right)$

b $P\left(\sum_{i=1}^{12} X_i > 1000\right) = P\left(\overline{X}_{12} > \frac{1000}{12}\right)$

$= P\left(\frac{\overline{X}_{12} - 75}{10/\sqrt{12}} > \frac{1000/12 - 75}{10/\sqrt{12}}\right)$

$= 1 - \Phi\left(\frac{5}{\sqrt{3}}\right) = 1 - \Phi(2.886) = 0{,}0019$

c. $EX_1 = \frac{1}{2}(a+b) = 75$

$\text{var } X_1 = \frac{1}{12}(b-a)^2 = 100$

$\therefore \quad a = 75 - 10\sqrt{3} \qquad b = 75 + 10\sqrt{3}$

$= 57{,}68 \qquad = 92{,}32$

d $P\left(\sum_{i=1}^{n} X_i > 1000\right) < 0{,}001$

$\Leftrightarrow \quad P\left(\frac{\sum_{i=1}^{n} X_i - 75n}{10\sqrt{n}} > \frac{1000-75n}{10\sqrt{n}}\right) < 0{,}001$

C.L.S. $\sim \quad 1 - \Phi\left(\frac{1000-75n}{10\sqrt{n}}\right) < 0{,}001$

$\Leftrightarrow \quad \Phi(\ldots) > 0{,}999$ 1

$\Leftrightarrow \quad \frac{1000-75n}{10\sqrt{n}} \geq 3{,}10$

$\Leftrightarrow \quad 31\sqrt{n} + 75n \leq 1000 \quad \Leftrightarrow \quad n \leq 11$

5 a $E\hat{p} = \frac{1}{n} EX = p \qquad var(\hat{p}) = \frac{1}{n^2} var(X) = \frac{1}{n} p(1-p)$

b Als T een schatter is van θ, dan is $E(T-\theta)^2$ de verwachte kwadratische fout van T

$E(\hat{p}-p)^2 = var(\hat{p}) = \frac{1}{n} p(1-p)$

c De breedte $2c\sqrt{\frac{1}{n}\hat{p}(1-\hat{p})}$ van het 95% BI voor p moet $\leq 0{,}02$ zijn. Hierin is $c = 1{,}96$ 1
n.l. $P(Z < c) = 0{,}975$
$\therefore \quad \sqrt{n} \geq 196\sqrt{\hat{p}(1-\hat{p})}$
hieraan is voldaan als $\sqrt{n} \geq \frac{1}{2} \cdot 196 \Leftrightarrow n \geq 9604$

6 a 95% BI : $(\bar{x} - cs/\sqrt{n}\ ,\ \bar{x} + cs/\sqrt{n})$

met $c = 2{,}13$ nl. $P(T_{15} \leq c) = 0{,}975$

$s = 3{,}80\ ,\ n = 16\ ,\ \bar{x} = 28{,}25$

$\therefore$

$(26{,}2265\ ,\ 30.2735)$

b. zelfde gegevens

95% VI : $\left(\bar{x} - cs\sqrt{1+\frac{1}{n}}\right), \bar{x} + cs\sqrt{1+\frac{1}{n}}\,)\,)$

$= (19{,}9069\ ,\ 36{,}5931)$

c. 1. model : steekproef $X_1, \ldots, X_{16}$ $X_i \sim N(\mu, \sigma^2)$

2. $H_0 : \mu \leq 26{,}5$ $H_1 : \mu > 26{,}5$

3. toetsingsgrootheid $T = \dfrac{\bar{X} - 26{,}5}{S/\sqrt{16}} \sim t_{15}$

4. $KG = [c, \infty]$ met

$P(T \geq c \mid H_0) = 0{,}05 \implies c = 1{,}75$

5. $t = \dfrac{28{,}25 - 26{,}5}{3{,}8/4} = 1{,}84$

6. $t \in KG$, dus H_0 wordt verworpen

7. De steekproefgegevens bevestigen de bewering van het bedrijf