Test Statistics for TCS/BIT (Module 6 -201800421) ,

Friday the 31st of January 2020, 8.45-11.00 h. Lecturer Dick Meijer, module-coordinator Randy Klaassen

This test consists of 5 exercises. The formula sheet and the probability tables are provided.
An ordinary scientific calculator is allowed, not a programmable one (GR).

1. A new interface for a smart (programmable) heating thermostat was designed by students: the aim was that users could intuitively program the weekly heating schedule without consulting the user`s guide.
30 potential users were asked to program a given schedule for the thermostat.
In the table below you find the ordered task completion times (TCT), in minutes.

2.28	2.29	2.29	2.41	2.44	2.45	2.56	2.62	3.05	3.21
3.22	3.26	3.32	3.37	3.44	3.46	3.88	4.28	4.35	4.42
4.54	4.55	4.57	5.05	5.09	5.13	5.29	5.83	5.97	7.34

a. Determine the 90th percentile of these measurements.

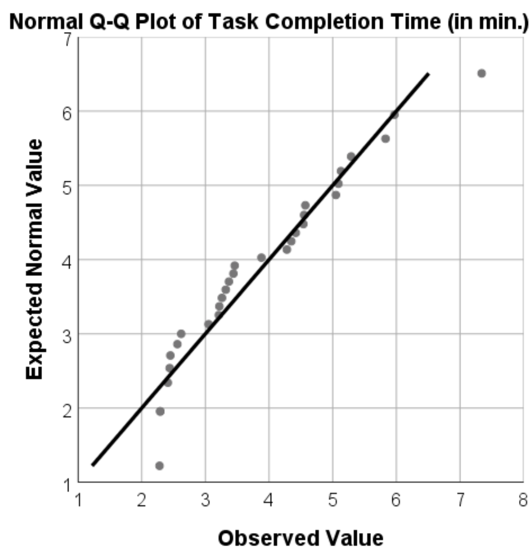
b. Determine the 5-number-summary of the observations and determine outliers, using the $1.5 \times IQR$-rule.

SPSS provided the following numerical summary (note that SPSS reports "**Kurtosis – 3**"), Shapiro Wilk`s test statistic and the normal Q-Q plot

Descriptive Statistics

	N	Mean	Std. Deviation	Variance	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Task Completion Time	30	3.8653	1.29561	1.679	0.715	0.427	0.121	0.833

Test on normality: Shapiro Wilk's $W = 0.929$



c. Can we assume a normal distribution for the observed task completion times?
Comment on:
1. The numerical summary,
2. The normal Q-Q plot and
3. The observed value of Shapiro Wilk's W

and draw your overall conclusion.

d. Assuming normality, give a 95%-confidence interval for the expected TCT and give the proper interpretation of the numerical interval.

2. An item of the Dutch 8 o`clock news on the 6th of August 2019 concerned the wearing of safety belts in cars. The presenter stated: "Traffic data revealed that last year (2018) 18 of the 58 deaths in car accidents did not wear a safety belt, whereas the year before (2017) only 11 out of 60 deaths did not wear a safety belt, although wearing a belt is compulsory." The interviewed official called the increase "substantial".

a. Is this increase also statistically significant at a 5% level? Conduct an appropriate test in 8 steps.

b. If we would test on the equality of the proportions against the inequality, a Chi-squared test is an alternative for the test, conducted in a.: give for this test (only) the test statistic and its observed value.

3. After some complaints about slow service in a fast food restaurant the management decided to compare the service times at this restaurant and another restaurant (of the same company) in the same town.
The observed 31 service times in the first restaurant were on average 80 seconds and the standard deviation was 8 seconds. In the second restaurant the mean of the 26 observed times was 73 seconds with a standard deviation of 6 seconds.

a. First test, with a 5% level of significance, whether assuming equal variances is allowed.
Only report: 1. The hypotheses
2. The test statistic and its observed value.
3. The rejection region
4. Your conclusion (in words).

b. Test whether there is a difference in mean service times between the two restaurants.
Use the appropriate parametric test (in 8 steps), with $\alpha = 5\%$.

c. Which non-parametric alternative would you apply for the test in b. ?
Give only the formula of the test statistic.

4. Is the (expensive) training for sales persons in a large insurance company effective?
Lately 10 of the sales persons were trained. For each of them the sales in a month time after the training were compared to the sales of the month before the training. The numbers of sold insurances were as follows:

Person	1	2	3	4	5	6	7	8	9	10
After	24	18	17	16	19	13	16	24	25	20
Before	22	19	16	15	16	12	15	24	23	18

The data analyst of the company advised **not** to use a parametric method (assuming normal distributions for the observations) to answer the question whether the training was effective.

a. Explain why you can support the choice of a non-parametric test in this case and which non-parametric test is appropriate.

b. Conduct the test in a. in 8 steps at a 5% level of significance.

c. Determine the power of the test in b. if in reality 80% of the sales persons improve their sales numbers.

5. A random number generator produces a random real number X with a unknown mean μ.
Assume that X has a uniform distribution on the interval $[0, 2\mu]$: then $E(X) = \mu$ and $var(X) = \frac{\mu^2}{3}$.
$X_1, \dots, X_n$ is a random sample of these numbers.

a. Show that $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is an unbiased estimator of μ.

b. Consider the family of estimators of μ given by $T = a\overline{X}$, where a is a positive real number.
For which value of a is T the best estimator of μ within this family?

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Grade $= 1 + \frac{\#\text{ points}}{46} \times 9$,
rounded at 1 decimal.

1				2		3			4			5		Tot
a	b	c	d	a	b	a	b	c	a	b	c	a	b	
1	4	4	4	6	2	4	6	1	2	6	3	1	2	46

Formula sheet "Statistics for Engineers"

Rules Probability Theory: $var(X) = E(X^2) - (EX)^2$
$E(aX + b) = aE(X) + b$ and $var(aX + b) = a^2 var(X)$
$E(X \pm Y) = E(X) \pm E(Y)$
If X and Y are independent: $var(X \pm Y) = var(X) + var(Y)$

Bounds for Confidence Intervals:

* $\hat{p} \pm c\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$, with $\Phi(c) = 1 - \frac{1}{2}\alpha$
* $\overline{X} \pm c\dfrac{S}{\sqrt{n}}$, with $P(T_{n-1} \geq c) = \frac{1}{2}\alpha$
* $\left(\dfrac{(n-1)S^2}{c_2}, \dfrac{(n-1)S^2}{c_1}\right)$, with $P(\chi^2_{n-1} \leq c_1) = P(\chi^2_{n-1} \geq c_2) = \dfrac{\alpha}{2}$
* $\overline{X} - \overline{Y} \pm c\sqrt{S^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}$, with $S^2 = \dfrac{n_1 - 1}{n_1 + n_2 - 2}S_X^2 + \dfrac{n_2 - 1}{n_1 + n_2 - 2}S_Y^2$
  and $P\left(T_{n_1+n_2-2} \geq c\right) = \frac{1}{2}\alpha$

  or: $\overline{X} - \overline{Y} \pm c\sqrt{\dfrac{S_X^2}{n_1} + \dfrac{S_Y^2}{n_2}}$, with $\Phi(c) = 1 - \frac{1}{2}\alpha$
* $\hat{p}_1 - \hat{p}_2 \pm c\sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$, with $\Phi(c) = 1 - \frac{1}{2}\alpha$

Testing procedure in 8 steps

1. Give a probability model of the observed values (the statistical assumptions).
2. State the null hypothesis and the alternative hypothesis, using parameters in the model.
3. Give the proper test statistic.
4. State the distribution of the test statistic if H_0 is true.
5. Compute (give) the observed value of the test statistic.
6. State the test and **a.** Determine the rejection region or
   b. Compute the p-value.
7. State your statistical conclusion: reject or fail to reject H_0 at the given significance level.
8. Draw the conclusion in words.

Test statistics and distributions under H_0:

* Binomial test: $X \sim B(n, p_0)$: $P(X = x) = \binom{n}{x}p_0^x(1-p_0)^{n-x}$ or use the binomial table,
  or for large n approximately $N\left(np_0, np_0(1-p_0)\right)$
* $T = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1}$

* S^2 , where $\dfrac{(n-1)S^2}{\sigma_0^2} \sim \chi^2_{n-1}$
* $T = \dfrac{(\overline{X}-\overline{Y})-\Delta_0}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim t_{n_1+n_2-2}$ (and S^2 as given above) or $Z = \dfrac{(\overline{X}-\overline{Y})-\Delta_0}{\sqrt{\frac{S_X^2}{n_1}+\frac{S_Y^2}{n_2}}} \sim N(0,1)$
* $Z = \dfrac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim N(0,1)$, with $\hat{p} = \dfrac{X_1+X_2}{n_1+n_2}$
* $F = \dfrac{S_X^2}{S_Y^2} \sim F^{n_1-1}_{n_2-1}$

Analysis of categorical variables

* 1 row and k columns: $\chi^2 = \sum_{i=1}^{k} \dfrac{(N_i - E_0N_i)^2}{E_0N_i}$ $(df = k-1)$
* $r \times c -$ cross table: $\chi^2 = \sum_{j=1}^{c}\sum_{i=1}^{r} \dfrac{\left(N_{ij}-\hat{E}_0N_{ij}\right)^2}{\hat{E}_0N_{ij}}$, with $\hat{E}_0N_{ij} = \dfrac{\text{row total} \times \text{column total}}{n}$

  and $df = (r-1)(c-1)$.

Non-parametric tests

* Sign test: $X \sim B\left(n, \frac{1}{2}\right)$ under H_0
* Wilcoxon`s Rank sum test: $W = \sum_{i=1}^{n_1} R(X_i)$,

  under H_0 with: $E(W) = \dfrac{1}{2}n_1(N+1)$ and $var(W) = \dfrac{1}{12}n_1n_2(N+1)$

Test on the normal distribution

* Shapiro – Wilk`s test statistic: $W = \dfrac{\left(\sum_{i=1}^{n} a_i X_{(i)}\right)^2}{\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2}$