## Exercises (based on questions 25-32 of exam 10 April 2017)

Consider the Petri net in the figure below.


If this is an Event/Condition ( $E / N$ ) net, its dynamic behavior is described by the following table.

| Marking | P1 | P2 | P3 | P4 | P5 | P6 | Enabled transitions |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| M0 | 1 | 0 | 0 | 1 | 0 | 0 | t2 -> M1, t5 -> M2 |
| M1 | 0 | 1 | 0 | 1 | 0 | 1 | t4 -> M3 |
| M2 | 1 | 0 | 0 | 0 | 0 | 1 | t4 -> M4 |
| M3 | 0 | 1 | 1 | 1 | 0 | 0 | t5 -> M5, t3 -> M6 |
| M4 | 1 | 0 | 1 | 0 | 0 | 0 | t2 -> M7, t3 -> M8 |
| M5 | 0 | 1 | 1 | 0 | 0 | 1 | t3 -> M9 |
| M6 | 0 | 1 | 0 | 1 | 1 | 0 | t5 -> M10, t1 -> M11 |
| M7 = M5 | 0 | 1 | 1 | 0 | 0 | 1 | Back to M5 |
| M8 | 1 | 0 | 0 | 0 | 1 | 0 | t2 -> M12, t6 -> M13 |
| M9 | 0 | 1 | 0 | 0 | 1 | 1 | t1 -> M14, t4 -> M15, t6 -> M16 |
| M10 = M9 | 0 | 1 | 0 | 0 | 1 | 1 | Back to M9 |
| M11 = M0 | 1 | 0 | 0 | 1 | 0 | 0 | Back to M0 |
| M12 = M9 | 0 | 1 | 0 | 0 | 1 | 1 | Back to M9 |
| $\mathrm{M} 13=\mathrm{M} 0$ | 1 | 0 | 0 | 1 | 0 | 0 | Back to M0 |
| M 14 = M2 | 1 | 0 | 0 | 0 | 0 | 1 | Back to M2 |
| M15 | 0 | 1 | 1 | 0 | 1 | 0 | t1 -> M17, t6 -> M18 |
| M16 = M1 | 0 | 1 | 0 | 1 | 0 | 1 | Back to M1 |
| $\mathrm{M} 17=\mathrm{M} 4$ | 1 | 0 | 1 | 0 | 0 | 0 | Back to M4 |
| $\mathrm{M} 18=\mathrm{M} 3$ | 0 | 1 | 1 | 1 | 0 | 0 | Back to M3 |

From the table you can see that there are never more than 3 tokens in the network (question 25). The maximum number of tokens is present in the states denoted by markings $\mathrm{M} 1, \mathrm{M} 3, \mathrm{M} 5, \mathrm{M} 6, \mathrm{M} 9$, and M15.

There are 10 unique markings in the table: $\mathrm{M} 0-\mathrm{M} 6, \mathrm{M} 8-\mathrm{M} 9$, and M 15 (question 26).
Some markings enable more than on transition. However, inspection of the table shows that shows that the following two transactions are never enabled at the same time (i.e., with the same marking): t1 and t2, t3 and t4, and t4 and t5 (question 27).

The maximum number of transitions enabled at the same time is 3 (question 27). This happens with marking M9.

If we treat the same net representation as an Place/Transition ( $P / T$ ) net, we have a different dynamic behavior. This behavior is described by the following table. (Extra markings and transitions compared to those possible with an E/C net are in red. The easiest way to make this table is by extending the previous table for the $E / C$ net: if for a particular marking of the $E / C$ net the number of tokens is greater than the number of enabled transitions, you should check whether the same marking of the $P / T$ net would enable an additional transition).

| Marking | P1 | P2 | P3 | P4 | P5 | P6 | Enabled transitions |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| M0 | 1 | 0 | 0 | 1 | 0 | 0 | t2 $->$ M1, t5 $->$ M2 |
| M1 | 0 | 1 | 0 | 1 | 0 | 1 | t4 $->$ M3, t5 $->$ M27 |
| M2 | 1 | 0 | 0 | 0 | 0 | 1 | t4 -> M4, t2 -> M18a |
| M3 | 0 | 1 | 1 | 1 | 0 | 0 | t5 -> M5, t3 -> M6 |
| M4 | 1 | 0 | 1 | 0 | 0 | 0 | t2 -> M7, t3 -> M8 |
| M5 | 0 | 1 | 1 | 0 | 0 | 1 | t3 -> M9, t4 -> M19 |
| M6 | 0 | 1 | 0 | 1 | 1 | 0 | t5 -> M10, t1 -> M11, t6 -> M20 |
| M7 = M5 | 0 | 1 | 1 | 0 | 0 | 1 | Back to M5 |
| M8 | 1 | 0 | 0 | 0 | 1 | 0 | t2 -> M12, t6 -> M13 |
| M9 | 0 | 1 | 0 | 0 | 1 | 1 | t1 -> M14, t4 -> M15, t6 -> M16 |
| M10 = M9 | 0 | 1 | 0 | 0 | 1 | 1 | Back to M9 |
| M11 = M0 | 1 | 0 | 0 | 1 | 0 | 0 | Back to M0 |
| M12 = M9 | 0 | 1 | 0 | 0 | 1 | 1 | Back to M9 |
| M13 = M0 | 1 | 0 | 0 | 1 | 0 | 0 | Back to M0 |
| M14 = M2 | 1 | 0 | 0 | 0 | 0 | 1 | Back to M2 |
| M15 | 0 | 1 | 1 | 0 | 1 | 0 | t1 -> M17, t6 -> M18, t3 -> M24 |
| M16 = M1 | 0 | 1 | 0 | 1 | 0 | 1 | Back to M1 |
| M17 = M4 | 1 | 0 | 1 | 0 | 0 | 0 | Back to M4 |
| M18 = M3 | 0 | 1 | 1 | 1 | 0 | 0 | Back to M3 |
| M18a | 0 | 1 | 0 | 0 | 0 | 2 | t4 -> M21 |
| M19 | 0 | 1 | 2 | 0 | 0 | 0 | t3 -> M22 |
| M20 | 0 | 1 | 0 | 2 | 0 | 0 | t5 -> M23 |
| M21 = M5 | 0 | 1 | 1 | 0 | 0 | 1 | Back to M5 |
| M22 = M15 | 0 | 1 | 1 | 0 | 1 | 0 | Back to M15 |
| M23 = M1 | 0 | 1 | 0 | 1 | 0 | 1 | Back to M1 |
| M24 | 0 | 1 | 0 | 0 | 2 | 0 | t1 -> M25, t6 -> M26 |
| M25 = M8 | 1 | 0 | 0 | 0 | 1 | 0 | Back to M8 |


| $\mathrm{M} 26=\mathrm{M} 6$ | 0 | 1 | 0 | 1 | 1 | 0 | Back to M6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{M} 27=\mathrm{M} 18 \mathrm{a}$ | 0 | 1 | 0 | 0 | 0 | 2 | Back to M18a |

Inspection of the table shows that:

- the maximum number of tokens in the network is 3 (question 29).
- the number of different markings is 14 (question 30 ).
- Transitions t 1 and t 2 are never enabled at the same time, but t 2 and $\mathrm{t} 3, \mathrm{t} 3$ and t 4 , and t 4 and t 5 can be enabled at the same time (question 31). You can immediately see that if you represent the enabled transitions per unique marking as follows:

|  | t 1 | t 2 | t 3 | t 4 | t 5 | t 6 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| M 0 |  | x |  |  | x |  |
| M 1 |  |  |  | x | x |  |
| M 2 |  | x |  | x |  |  |
| M 3 |  |  | x |  | x |  |
| M 4 |  | x | x |  |  |  |
| M 5 |  |  | x | x |  |  |
| M 6 | x |  |  |  | x | x |
| M 8 |  | x |  |  |  | x |
| M9 | x |  |  | x |  | x |
| M15 | x |  | x |  |  | x |
| M18a |  |  |  | x |  |  |
| M19 |  |  | x |  |  |  |
| M20 |  |  |  |  | x |  |
| M24 | x |  |  |  |  | x |

- The maximum number of transitions is 3 (question 32 ).

