Course : **Discrete Mathematics for Technical Computer Science; Part 1**
Date : January 14, 2021
Time : 09.00-10.00 hrs

Motivate all your answers.
The use of electronic devices is not allowed.
A formula sheet is included.
Write your solutions of Part 1 (Questions 1, 2 and 3) and Part 2 (Questions 4, 5 and 6) on two separate sheets of paper!

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

1. [6 pt]
   Let A, B and C be sets in a universe $\mathcal{U}$. Give quantified expressions for the following statements.

   (a) $A \cap B = \emptyset$

   (b) C is a subset of A but C is not a subset of B.

2. [6 pt]

   Prove the validity of the following argument using the "Laws of Logic" and the "Rules of Inference".

$$\begin{array}{l} u \to r \\ (r \wedge s) \to (p \vee t) \\ q \to (u \wedge s) \\ \neg t \\ q \\ \hline \therefore p \end{array}$$

3. [6 pt]
   Let A and B be sets in a universe $\mathcal{U}$. Give a proof or a counterexample for the following equality:

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

Total: 18 points

Course : **Discrete Mathematics for Technical Computer Science; Part 2**
Date : January 14, 2021
Time : 10.00–11.00 hrs

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A formula sheet is included.
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In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

4. [6 pt]
   Let $a_0 = 0$, $a_1 = 1$ and let a_n for $n \geq 2$ be defined by:

   $a_n = \frac{1}{n} \cdot a_{n-1} + \frac{n-1}{n} \cdot a_{n-2}$

   Prove for all $n \in \mathbb{N}$ for which $n \geq 2 : 0 \leq a_n \leq 1$

5. [6 pt]
   Let A and B be sets, $f : A \to B$ a function and $A_1, A_2 \subseteq A$.

   Prove that if f is one-to-one, $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$.

6. [6 pt]
   Let $A = \mathbb{R}$ and let R be the relation on A given by:

   $$(x, y) \in R \quad \text{if and only if} \quad |x - y| \in \mathbb{Z} \text{ i.e. } |x - y| \text{ is an integer.}$$

   (a) Show that R is an equivalence relation on A.

   (b) Give a description of $[1.5]$ i.e. the equivalence class of 1.5 .

Total: 18 points

Laws of Logic

L1.	$\neg\neg p \Leftrightarrow p$	Law of Double Negation
L2.	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	DeMorgan's Laws
L3.	$p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	Commutative Laws
L4.	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$ $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	Associative Laws
L5.	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributive Laws
L6.	$p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	Idempotent Laws
L7.	$p \vee F_0 \Leftrightarrow p$ $p \wedge T_0 \Leftrightarrow p$	Identity Laws
L8.	$p \vee \neg p \Leftrightarrow T_0$ $p \wedge \neg p \Leftrightarrow F_0$	Inverse Laws
L9.	$p \vee T_0 \Leftrightarrow T_0$ $p \wedge F_0 \Leftrightarrow F_0$	Domination Laws
L10.	$p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	Absorption Laws
L11.	$p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$	
L12.	$p \rightarrow q \Leftrightarrow \neg p \vee q$	
L13.	$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$	

Rules of Inference

R1. Modus Ponens

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

R2. Law of the Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

R3. Modus Tollens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

R4. Rule of Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

R5. Rule of Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

R6. Rule of Contradiction

$$\begin{array}{l} \neg p \to F_0 \\ \hline \therefore p \end{array}$$

R7. Rule of Conjunctive Simplification

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$$

R8. Rule of Disjunctive Amplification

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

R9. Rule of Conditional Proof

$$\begin{array}{l} p \wedge q \\ p \to (q \to r) \\ \hline \therefore r \end{array}$$

R10. Rule for Proof by Cases

$$\begin{array}{l} p \to r \\ q \to r \\ \hline \therefore (p \vee q) \to r \end{array}$$

R11. Rule of the Constructive Dilemma

$$\begin{array}{l} p \to q \\ r \to s \\ p \vee r \\ \hline \therefore (q \vee s) \end{array}$$

R12. Rule of the Destructive Dilemma

$$\begin{array}{l} p \to q \\ r \to s \\ \neg q \vee \neg s \\ \hline \therefore \neg p \vee \neg r \end{array}$$

Laws concerning quantifiers

N1. $\neg[\forall x\, p(x)] \;\Leftrightarrow\; \exists x\, \neg p(x)$

N2. $\neg[\exists x\, p(x)] \;\Leftrightarrow\; \forall x\, \neg p(x)$

Additional Laws concerning quantifiers

U1. $$\frac{\forall x\, p(x)}{\therefore\, p(c) \text{ for arbitrary } c \text{ in the universe}}$$

U2. $$\frac{\exists x\, p(x)}{\therefore\, p(c) \text{ for some } c \text{ in the universe}}$$

U3. $$\frac{p(c) \text{ for arbitrary } c \text{ in the universe}}{\therefore\, \forall x\, p(x)}$$

U4. $$\frac{p(c) \text{ for some } c \text{ in the universe}}{\therefore\, \exists x\, p(x)}$$

U1: Rule of Universal Specification

U2: Rule of Existential Specification

U3: Rule of Universal Generalization

U4: Rule of Existential Generalization