UNIVERSITEIT TWENTE
Faculteit Elektrotechniek, Wiskunde en Informatica

Exam Systems and Signals (201200111) on Friday, November 2, 2012, 8.45 – 11.45 pm.

The answers to the questions need to be clearly formulated and written down neatly. Moreover, in all cases you need to motivate your answer by including a derivation.

One page of handwritten notes can be used during the exam. A computer can be used during the exam. However, it amounts to fraud if any software is running actively or in the background which is aimed at communication with other people: E-mail, twitter, SMS, IM, etc.

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1. We have:

$$3\sin\left(\omega t - \tfrac{\pi}{3}\right) - \cos\left(\omega t + \tfrac{9\pi}{4}\right) = a\sin(\omega t) + b\cos(\omega t)$$

Determine the constants a and b.

2. Consider the functions

$$f(t) = \begin{cases} 1 + |t| & \text{for } |t| < 1 \\ 0 & \text{for } |t| > 1 \end{cases}$$

for which it is known that the Fourier transform of $f(t)$ is given by:

$$\hat{a}(\omega) = \frac{4\omega \sin\omega - 2\sin^2\omega}{\omega^2}, \qquad \hat{b}(\omega) = 0,$$

a) Draw the graph of the function $f(t)$ and its derivative.

b) Determine the Fourier transform of the following function:

$$g(t) = 2f'(2t)$$

3. Load `ex3.mat`. We have a system with input $u(t)$ given by `u` and associated time `t` which results in an output given by `y` (with the same associated time `t`. Argue whether you find this consistent with a linear, time-invariant system.

4. Load `ex4.mat`. We have a signal $x(t)$ given by `x` and associated time `t`. This signal is a combination of sinusoids.

a) Determine how many sinusoids you need to describe the signal and determine their frequencies $\omega_1, \dots, \omega_k$.

b) Determine coefficients $a_1, \dots, a_k$ and $b_1, \dots, b_k$ such that the signal can be described by:

$$a_1\cos(\omega_1 t) + \cdots + a_k\cos(\omega_k t) + b_1\sin(\omega_1 t) + \cdots + b_k\sin(\omega_k t)$$

Verify whether the signal is indeed accurately described by the above expression.

see reverse

5. We are looking at the unemployment in the U.S.A. between 1948 and 2001. These are monthly figures in 1000's of people. Load `ex5.mat`. We have a signal $z(t)$ given by `z` and associated time `t`.

   a) Check whether the signal contains a trend. If the signal contains a trend argue whether a linear, piecewise linear or polynomial trend is best. Compute the signal after the trend has been removed.

   b) Determine the Fourier transform of the signal (without trend) and plot the amplitude Fourier transform. Does this signal contain a periodic component and, if yes, what is the frequency.

   c) Using the command `periodic` to remove the periodic component from the signal (without trend). Comment on how well the periodic component has been removed.

6. We consider the following picture:

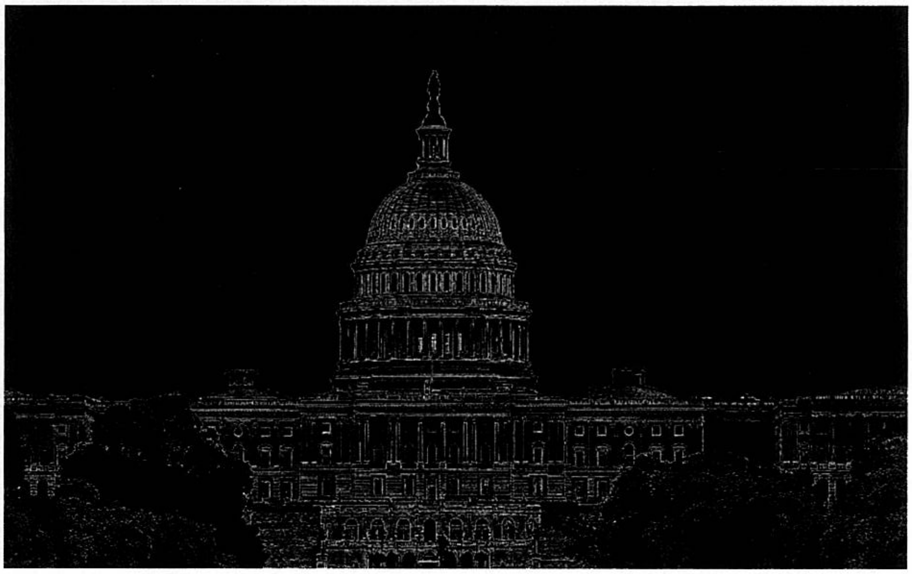
and the following 4 filters:

$$H_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$H_3 = \begin{pmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{pmatrix}, \quad H_4 = \begin{pmatrix} -1 & -1 & -1 \\ -1 & 13 & -1 \\ -1 & -1 & -1 \end{pmatrix}$$

We get the following results after convolution with the above filters:

Result 1:



Result 2:

Result 3:

Result 4:

Argue which filter belongs to which result.

For the questions the following number of points can be awarded:

Exercise	1.	5 points	Exercise	4.	6 points
Exercise	2.	5 points	Exercise	5.	7 points
Exercise	3.	6 points	Exercise	6.	7 points

The final grade is determined by adding 4 points to the total number of points awarded and dividing by 4.