Meike

Exam Part 2, Web Science 201500025 (Games, Auctions, Voting)

Friday, January 15, 2016, 8.45-11.45

- Use of calculators, mobile phones, etc. is not allowed!
- This exam consists of five problems. Please start a new page for every problem.
- You have 3 h time.
- Please write name and student ID on your solutions.
- Total number of points: 36+4 = 40. Distribution of points:

1a: 2	2a: 3	3a: 3	4a: 2	5a: 3
1b: 2	2b: 3	3b: 3	4b: 2	5b: 3
1c: 2	:	3c: 2	4c: 1	5c: 5

Question 1

Decide for each of the following statements whether it is true or false. Give a short argument to justify your answer (one or two sentences, or a counterexample). *(2 points per statement)*

(a) For every two-player game, the following holds: If the game possesses a pure Nash equilibrium, then at least one of the players has a dominating strategy.

(b) For every two-player game, the following holds: If both players have strictly dominating strategies, then there is exactly one pure Nash equilibrium.

(c) In every *first*-price sealed-bid auction, the following holds: If all bidders bid truthfully, then all bidders have a revenue of 0.

Question 2

Consider the game specified by the following payoff matrix and answer the following questions.

		opponent ℓ	m	r
	T	6/4	0/5	4/4
you	M	7/3	1/1	5/2
	B	9/2	3/2	2/1

(a) *(3 points)* Which strategies are strictly dominated by which strategies? Write down the resulting game with the strictly dominated strategies removed.

Does the resulting reduced game have strictly dominated strategies? If yes, by which strategies are they dominated? Iterate the process of removing strictly dominated strategies until no strictly dominated strategy remains.

(b) *(3 points)* Does the resulting game have pure Nash equilibria? If yes, list all pure Nash equilibria. Does the game have mixed Nash equilibria? If yes, compute and describe them.

Question 3

Consider the following example of a sponsored search auction with advertisers X, Y, and Z and three slots.

		10	5	0	clickthrough rate
		slot 1	slot 2	slot 3	
8	X	80	40	0	
6	Y	60	30	0	
4	Z	40	20	0	
value					

(a) *(3 points)* Compute market-clearing prices and a corresponding assignment of advertisers to slots together with their payoffs for this matching market by raising prices according to the price-raising procedure.

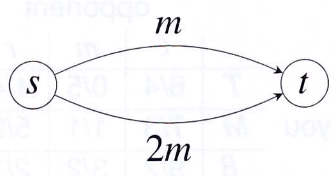
(b) *(3 points)* Run the VCG mechanism to compute an assignment of slots to advertisers and corresponding prices. You only have to compute the relevant personalized prices.

Draw the preferred-seller graph for the corresponding prices (with the price paid as the price for everybody).

(c) *(2 points)* Run a generalized second-price auction for this sponsored search auction. Assume that all three advertisers bid truthfully their values. What are the advertisers' payoffs?

Question 4

Consider the following simple road network. There are 99 cars that want to go from s to t. The travel time on the upper road is always m minutes if there are m cars on that road. The travel time on the lower road is $2m$ minutes if there are m cars on that road.



(a) *(2 points)* Compute the social optimum for this road network.

(b) *(2 points)* Compute a Nash equilibrium for this road network.

(c) *(1 point)* What can you conclude for the price of anarchy for this network routing game?

Question 5

(a) *(3 points)* Consider a voting rule on $m \geq 3$ alternatives in which voters with complete and transitive preferences distribute the scores 5, 3 and 1 over their three most favoured alternatives.

Prove or give a counterexample: If there is a Condorcet winner, then this alternative wins.

(b) *(3 points)* Suppose that you receive the assignment to design a voting rule F that fulfils two requirements: it should be unanimous, meaning that if $X \succ_i Y$ for all voters i in a preference profile P, then $X \succ Y$ in $F(P)$, and it should also be anonymous, meaning that $F(\succ_1, \ldots, \succ_n) = F(\succ_{\pi(1)}, \ldots, \succ_{\pi(n)})$ for any permutation π of the voters. We assume for simplicity that the number of voters is odd.

Give a voting rule that fulfils both requirements. Also give at least one major weakness of *any* voting rule that fulfils these two requirements.

(c) *(5 points)* Consider the following experiment. There are two decks, one with 10 red cards, and one with 1 red and 9 black cards. Let us call the first deck red, and the second black. Any of the two decks is chosen with probability $\frac{1}{2}$. Once that has been done, there are 3 players who get to see, each individually, one random card of the deck. In order to decide if the deck is actually red or black, the players must vote red or black *by majority*. Assume players are non-strategic (i.e., they follow their private signals).

(i) What is the probability that a player sees a red card?

(ii) What is the probability that the players decide red? (Hint: the result is $> \frac{1}{2}$.)

(iii) Suppose the players have decided red. What is the conditional probability that the deck is indeed red?

(iv) Briefly discuss: When a player sees a red card, and is interested in the correct outcome, should he actually be non-strategic and follow the signal?

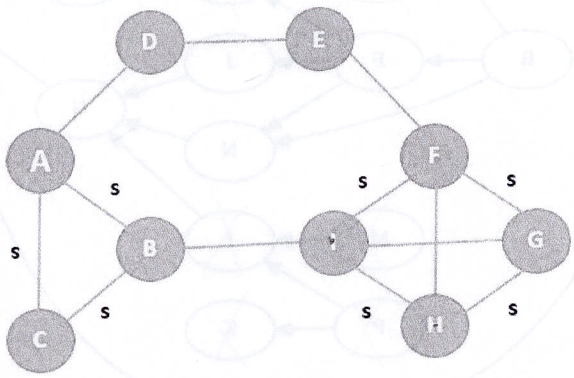
Meile

First Partial Exam Web Science

9th December 2015

Question 1, Social Networks

Consider the graph in the figure below.



a) Based on the existing connections, which connections are more likely to appear next? Motivate your answer.

b) Does the network on the figure contains a bridge? Which ties form a local bridge?

c) A *diameter* of a graph is the longest distance in a graph. What is the diameter of the graph in the figure?

In d), e), assume that the edges marked with the letter 's' represent strong ties.

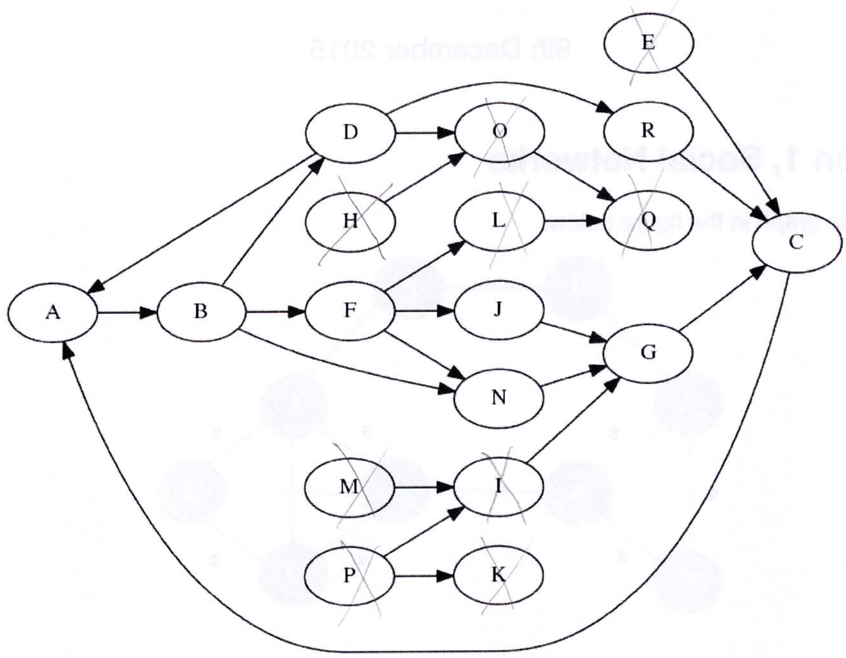
d) Assume that each node in the network satisfies the Strong Triadic Closure property. Which of the currently unmarked ties are definitely weak ties? Explain your answer.

e) Do local bridges always represent weak ties? Do all weak ties have to be local bridges? Explain your answers.

(Question 2 on next page...)

Question 2, The Structure of the Web

Consider the directed graph with a portion of a web crawl, where the nodes are web pages, and the edges are hyperlinks from one page to another.



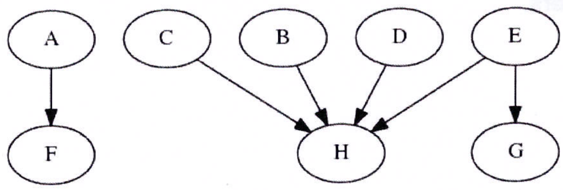
Answer the following questions, using the analysis and the definitions of Andrei Broder et al. “Graph Structure in the Web”, Proceedings of the 9th WWW Conference, 2000.

a) Which set of nodes constitutes the largest strongly connected component (SCC) in this graph? Explain your answer.

b) Taking the answer of a) as the giant SCC, which nodes belong to the sets IN and OUT? Explain your answer.

c) Which nodes belong to the tendrils? Explain your answer.

Consider the following bipartite graph that models the top 5 results of a web search query for “hotels”, where nodes are web pages and edges are links between web pages.

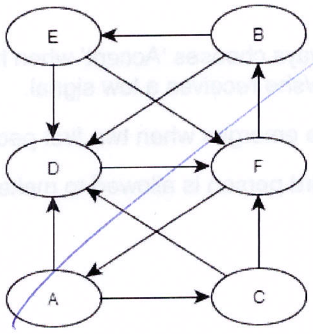


Answer the following questions:

d) Show the values that you get if you run two rounds of computing hubs and authority value updates on the graph of Web pages (so, k-step hub-authority computation with $k = 2$). Take 1 as the initial value of each page. Do not normalize the computed values.

e) Show the values after two rounds of computing hubs and authority value updates including a normalization step (It is fine to write the normalized scores as fractions rather than decimals.)

Consider the following directed graph that models a portion of a web crawl, where the nodes are web pages, and the edges are hyperlinks from one page to another. The graph also shows the proposed PageRank value for each one, expressed as a decimal next to the node.



Answer the following questions by applying the basic PageRank update rule, so the definition of PageRank that does not use scaling (or a scaling factor $s = 1$):

f) Does the assignment of numbers to the nodes from an equilibrium set of PageRank values for this graph? Explain your answer.

g) Suppose we apply the basic PageRank algorithm to the network of Question a). What would be the PageRank of each node in the graph? Explain your answer.

(Question 3 on next page...)

Question 3, Network Dynamics: Population Models

We consider the information cascade model of Chapter 16 with specific values for the probabilities as follows: the probability that Accept (A) is a good idea is p=1/2, and the probability of a High signal if Good is true (as well as the probability of a Low signal if Bad is true) is q=4/5. According to the model, a person chooses 'Accept' if, given the information he/she has, the probability that Good is true is greater than 1/2. Further, let's assume that Good is actually true.

a) Assume the first person has seen a high signal. Then, according to this person, what is the probability that Good is true?

b) Show that the first person always chooses 'Accept' when he/she receives a high signal and always chooses 'Reject' is he/she receives a low signal.

c) Show that the Reject cascade emerges when two first people choose Reject.

d) What changes in (c), if the third person is allowed to make two observations?

(Question 4 on next page...)

Question 4, Network Dynamics: Structural Models

Question 4.1

a) Explain what is a power law distribution of the degrees in a network. How do we recognize power laws? Why this model can be used to formally describe the presence of hubs – nodes with large number of connections?

b) A researcher is searching for a relevant scientific literature using *Google Scholar*, which shows the number of citations received by each paper. Explain how this may lead to the power law distribution in the number of citations across scientific papers.

Question 4.2

A clique of size k is a sub-graph, which contains k nodes, all connected to each other. Consider a network, which contains a clique of size k such that only one node v in this clique is connected to another node w outside of the clique. Consider the model from Chapter 19 for the diffusion of a new behavior through a social network. Everyone starts with behavior B, and a threshold for switching to a new behavior A is q. Any node will switch to A if at least a fraction q of its neighbors has adopted A. Assume that node w has switched to A.

a) Take $k = 4$. For which value of q no node in the clique switches to A?

b) Generalize the result in (a) to a clique of an arbitrary size $k \geq 3$.

c) Consider node $x \neq v$ is in the clique. Assume that x has no connections outside the clique. Is it possible that x switches to A while some other nodes in the clique stick to B?

d) What changes if the nodes in the clique create more links to nodes, which accepted A, outside the clique? Consider several scenarios. For example, is it possible that only some of the clique members switch to A while others stick to B?