Kenmerk : TW2018/DWMP/001/ha

Course : **Discrete Mathematics for Technical Computer Science; Part 1**
Date : October 26, 2018
Time : 08.45–09.45 hrs

Motivate all your answers. The use of electronic devices is not allowed.
A formula sheet is included.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

1. [6 pt]
   Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Give quantified expressions for the following statements.

   (a) f is decreasing.

   (b) The minimum value of f is equal to 0 and the maximum value of f is equal to 5.

2. [6 pt]
   Prove the validity of the following argument using the "Laws of Logic" and the "Rules of Inference".

$$\begin{array}{l} \neg q \vee s \\ (p \to q) \wedge r \\ \underline{(p \to r) \to p} \\ \therefore \neg(s \to \neg p) \end{array}$$

3. [6 pt]
   Let A, B and C be sets in a universe $\mathcal{U}$. Give a proof or a counterexample for each of the following statements.

   (a)
$$(A - C = B - C \quad \wedge \quad A \cup C = B \cup C) \quad \Longrightarrow \quad A = B.$$

   (b)
$$(A - C = B - C \quad \wedge \quad A \cap C = B \cap C) \quad \Longrightarrow \quad A = B.$$

Total: 18 points

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Course : **Discrete Mathematics for Technical Computer Science; Part 2**
Date : October 26, 2018
Time : 09.45–10.45 hrs

Motivate all your answers. The use of electronic devices is not allowed.
A formula sheet is included.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

4. [6 pt]
   Let the sequence of numbers $a_0, a_1, a_2, a_3, \ldots$ be given by:
   $a_0 = 1$, $a_1 = 1$, $a_2 = 2$, and for $n \geq 3$: $\quad a_n = a_{n-2} + a_{n-3}$.
   Prove with mathematical induction that for all $n \in \mathbb{N}$:
   $$a_n \leq \left(\sqrt{2}\right)^n.$$

5. [6 pt]
   Let f be the closed binary operation on $\mathcal{P}(\{1,2,3\})$ given by
   $$f(A, B) = A - B \qquad (A, B \subseteq \{1,2,3\})$$
   (a) Determine if f is one-to-one; if f is onto; and if f is invertible.
   (b) Determine $\quad f^{-1}(\{1,3\})$.

6. [6 pt]
   Let D be a set in a universe $\mathcal{U}$. Define the relation R on $\mathcal{P}(\mathcal{U})$ by:
   $$ARB \quad \text{if and only if} \quad A - D = B - D \qquad (A, B \subseteq \mathcal{U}).$$
   (a) Show that R is an equivalence relation on $\mathcal{P}(\mathcal{U})$.
   (b) Take $\mathcal{U} = \{1,2,3,4\}$ and $D = \{1,2\}$. Determine the partition of $\mathcal{P}(\mathcal{U})$ induced by R.

Total: 18 points

Rules of Inference

R1. p, $p \to q$ $\therefore q$ — Modus Ponens

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

R2. Law of the Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

R3. Modus Tollens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

R4. Rule of Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

R5. Rule of Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

R6. Rule of Contradiction

$$\begin{array}{l} \neg p \to F_0 \\ \hline \therefore p \end{array}$$

R7. Rule of Conjunctive Simplification

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$$

R8. Rule of Disjunctive Amplification

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

R9. Rule of Conditional Proof

$$\begin{array}{l} p \wedge q \\ p \to (q \to r) \\ \hline \therefore r \end{array}$$

R10. Rule for Proof by Cases

$$\begin{array}{l} p \to r \\ q \to r \\ \hline \therefore (p \vee q) \to r \end{array}$$

R11. Rule of the Constructive Dilemma

$$\begin{array}{l} p \to q \\ r \to s \\ p \vee r \\ \hline \therefore (q \vee s) \end{array}$$

R12. Rule of the Destructive Dilemma

$$\begin{array}{l} p \to q \\ r \to s \\ \neg q \vee \neg s \\ \hline \therefore \neg p \vee \neg r \end{array}$$

Laws of Logic

L1.	$\neg\neg p \Leftrightarrow p$	Law of Double Negation
L2.	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	DeMorgan's Laws
L3.	$p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	Commutative Laws
L4.	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$ $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	Associative Laws
L5.	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributive Laws
L6.	$p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	Idempotent Laws
L7.	$p \vee F_0 \Leftrightarrow p$ $p \wedge T_0 \Leftrightarrow p$	Identity Laws
L8.	$p \vee \neg p \Leftrightarrow T_0$ $p \wedge \neg p \Leftrightarrow F_0$	Inverse Laws
L9.	$p \vee T_0 \Leftrightarrow T_0$ $p \wedge F_0 \Leftrightarrow F_0$	Domination Laws
L10.	$p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	Absorption Laws
L11.	$p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$	
L12.	$p \rightarrow q \Leftrightarrow \neg p \vee q$	
L13.	$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$	

Laws concerning quantifiers

N1. $\neg[\forall x\, p(x)] \;\Leftrightarrow\; \exists x\, \neg p(x)$

N2. $\neg[\exists x\, p(x)] \;\Leftrightarrow\; \forall x\, \neg p(x)$

Additional Laws concerning quantifiers

U1. $$\frac{\forall x\, p(x)}{\therefore\, p(c) \text{ for arbitrary } c \text{ in the universe}}$$

U2. $$\frac{\exists x\, p(x)}{\therefore\, p(c) \text{ for some } c \text{ in the universe}}$$

U3. $$\frac{p(c) \text{ for arbitrary } c \text{ in the universe}}{\therefore\, \forall x\, p(x)}$$

U4. $$\frac{p(c) \text{ for some } c \text{ in the universe}}{\therefore\, \exists x\, p(x)}$$

U1: Rule of Universal Specification

U2: Rule of Existential Specification

U3: Rule of Universal Generalization

U4: Rule of Existential Generalization