UNIVERSITY OF TWENTE.

Course	:	Statistical Techniques for CS/BIT
Module	:	M6 Intelligent Interaction Design
Course code	:	202001033
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Time	:	13:45 – 16:00 (2hrs 15min)

Statistical Techniques for CS/BIT

Final Test

Instructions

- This test consists of 7 exercises
- All your answers must be submitted on the **answer sheet** provided
- Additional scratch paper is available for your convenience (not graded!)
- The formula sheet and the probability tables are provided separately
- An ordinary calculator is allowed, not a programmable one (GR)

1. After rolling out the optical fibre network in a region, an internet provider offers a standard subscription that provides a stable download speed of 100 Mbit/s. When asked what "stable" means, the provider stated that the expected speed for customers is at least 90 Mbit/s and the standard deviation of the speeds of customers is at most 5 Mbit/s. A customer organisation checks these claims using data provided by a random sample of 20 customers. All observations were measured under similar circumstances. Below the observations are ordered and summarized numerically.

75.1	76.3	80.4	82.6	83.7	84.1	86.8	87.2	87.9	88.0
89.0	89.6	90.2	91.0	91.1	93.3	94.7	95.3	98.7	102.2

Numerical summary:

Sample Size	Mean	Std. Dev.	Variance	Skewness	SE Skewness	Kurtosis*	SE Kurtosis
20	88.36	6.855	46.99	-0.082	0.512	0.084	0.992

*Kurtosis is adjusted as in SPSS (kurtosis − 3).

a. Determine the 5-number-summary and check whether there are outliers using the $1.5 \times \text{IQR}$ rule.

b. Comment whether the normality assumption seems to hold. Justify your arguments using both the numerical summary, and the information you gained from part **a.**

c. Statistical software reports a value of $W = 0.985$ for Shapiro-Wilk's test on normality. What can you conclude from this value? Justify your reasoning.

2. We continue to discuss the data from Question 1. The provider claims that "the expected speed is at least 90 Mbit/s". The consumer agency suspects this is not the case, and wants to check whether this claim is valid. Does the data support the agency's suspicion?

   a. Which statistical test can be correctly applied here to answer the above question? Provide your final answer in each of the following two situations:

   (1) If the data can be assumed normal.

   (2) If the data cannot be assumed normal.

   b. Apply the test procedure (provide all 8 steps) for the test you selected in **a.**(1) to answer the question with $\alpha = 5\%$.

   c. For the test you selected in **a.**(2) provide:

   (1) The assumptions needed for the test you chose.

   (2) The p-value of the test.

   (3) Your conclusion (for $\alpha = 5\%$) about the provider's claim.

3. Let X_1,X_2 and X_3 be a random sample of observations from a population with mean μ and variance σ^2. Consider the following estimators of μ : $\theta_1 = \frac{1}{2}X_1 + \frac{1}{3}X_2 + \frac{1}{6}X_3$ and $\theta_2 = \frac{1}{5}X_1 + \frac{2}{5}X_2 + \frac{3}{5}X_3$. Which of the following statements is (are) true? (Check all that apply).

   (a) The variance of θ_1 is σ^2

   (b) The variance of θ_1 is $\frac{14}{36}\sigma^2$

   (c) The variance of θ_2 is $\frac{6}{5}\sigma^2$

   (d) The variance of θ_2 is $\frac{14}{25}\sigma^2$

   (e) θ_1 is an unbiased estimator

   (f) θ_2 is an unbiased estimator

   (g) θ_1 is a better estimator than θ_2

4. Last year, during the corona crisis, about 60% of bachelor graduates from the EEMCS faculty continued their study with a master program at the UT. The faculty management is worried that this proportion will drop as a result of the increased opportunities of studying abroad at the post-Corona stage. A group of 100 bachelor students in their third year were surveyed about their preferences to continue their studies in a master program at the UT if travel restrictions are lifted: 50 of them said they would.

   a. If we consider the above group of 100 students to be a random sample of all future EEMCS bachelor graduates, construct a 95%–confidence interval for the proportion of bachelor students who will continue their study in a master program at the UT.

   b. Consider the width of the interval in **a.** How large should we choose the sample size in order to create a 95%–confidence interval with a width (length) of at most 0.04?

5. Medical investigators have developed a new artificial heart constructed primarily of titanium and plastic. This artificial heart will last and operate almost indefinitely once it is implanted in the patient's body, though the battery pack needs to be recharged about every 4 hours. A random sample of 50 battery packages is selected and subjected to a life test. It was observed that the average life of these batteries is 4.05 hours. From previous tests, the investigators know that the life of these batteries follows a normal distribution and that the standard deviation is $\sigma = 0.2$ hours.

   a. If the investigators want to test whether the mean battery life exceeds 4 hours (with $\alpha = 5\%$), what is the correct test to apply?

   b. Compute the power of the above test if the true mean battery life is 4.15 hours.

6. The author of a popular book on a diet method claims that the timing of meals influences the effect of the diet:

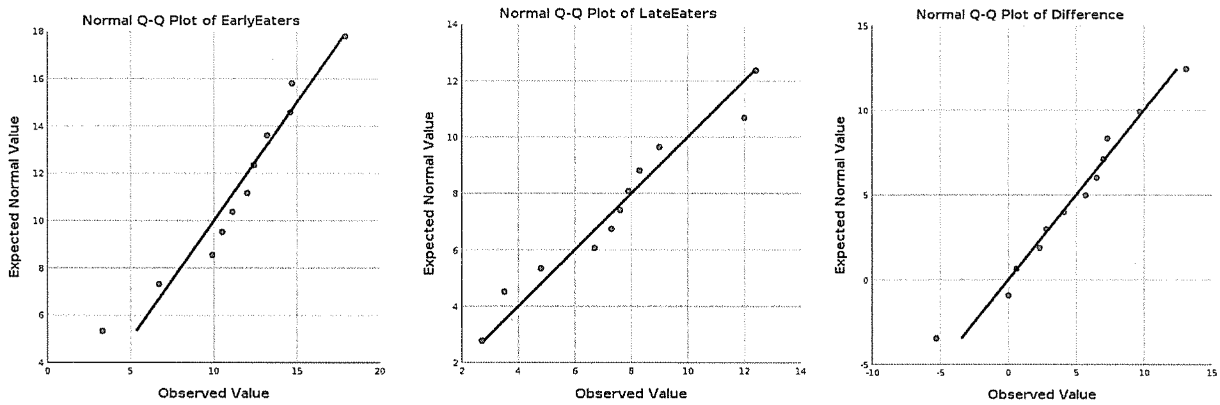
"The earlier one eats during the day, the larger the weight loss will be". $(*)$

Since no research results for such a claim are available, researchers decided to conduct a preliminary study on the topic for relatively small random samples of obese individuals. One group consists of "Early eaters" and the other of "Late eaters". Below are the weight losses (in kg) after a 20-week treatment:

Subject #	1	2	3	4	5	6	7	8	9	10	11	12
Early eaters	10.5	13.2	12.4	12.4	14.6	3.3	12.0	14.7	11.1	17.9	6.7	9.9
Late eaters	3.5	6.7	2.7	12.4	7.3	2.7	7.9	9.0	8.3	4.8	12.0	7.6
Difference	7.0	6.5	9.7	0.0	7.3	0.6	4.1	5.7	2.8	13.1	-5.3	2.3

The researchers have obtained the following output from statistical software:

	Sample Size	Mean	Std. Dev.
Early eaters	12	11.56	3.81
Late eaters	12	7.08	3.24
Difference	12	4.48	4.86



F Test for Equality of Variances

		d.f.	F statistic	p-value
Groups	Early eaters	11	.057	.814
	Late eaters	11		

a. Is there a statistical test (covered in this course) that can be correctly applied here to decide whether the weight loss is larger for early eaters?

(1) If your answer is yes: justify your argument by explicitly referring to the assumptions of the test, and state the test statistic.

(2) If your answer is no: provide the reasons to justify your argument.

b. Let μ denote the difference as follows: expected weight loss for early eaters minus expected weight loss for late eaters. A 95% confidence interval for μ has been computed by the researchers and reported as

$$(0.106, 7.577).$$

(In this part, you may assume that the researchers have applied *some* method for which none of the relevant assumptions was violated and the procedure has been correctly executed.)

Does it follow from the above confidence interval that the weight loss for early eaters is significantly different from that for late eaters (at a 5% significance level)? Does this prove the author's claim as stated above (marked above by (*))? Why (not)?

c. With respect to the interval in part **b.**, is it correct to state: $\mathbf{P}(0.106 \leq \mu \leq 7.577) = 0.95$? Explain your answer.

7. A psychologist studies the perception of children towards their parent's work/life balance. For this purpose, the psychologist designs a survey to ask children whether they think that their parents work too much. In order to conduct the survey, the psychologist randomly selects 10 houses and polls all the children living in each house. From these 10 houses, the psychologist obtains a total of $n = 21$ responses.

After getting the results from the poll, the psychologist estimates that two-thirds of the children perceive that their parents work too much. A binomial test is carried out (by *binomial test* we mean a test on a population proportion where the test statistic is a binomial random variable). The psychologist tests H_0: the true proportion of children with such perception is at most 50% vs H_1: the true proportion is larger than 50%. After careful computation, the p-value of this test is found to be 0.0946. (You may assume this computation was correctly executed).

The psychologist claims that the data shows enough evidence to conclude that most children perceive that their parents work too much (at a significance level of 10%). Is this claim correctly justified?

a. Why do you think the researcher chooses a binomial test instead of a t-test or a Z-test? Justify your answer.

b. List all the assumptions in a binomial test and judge whether or not they are satisfied in this particular case.

c. Can the psychologist's claim be concluded from the test? Explain your answer.

$$\text{Grade} = 1 + \frac{\#\text{ points}}{40} \times 9$$

Rounded to 1 decimal

question	1	2	3	4	5	6	7	Total
points possible	8	9	2	5	6	6	4	40