

**Discrete Mathematics for Computer Science**  
Sample Test, Part 2

**Duration: 60 min.**

**Motivate all your answers.**

**The use of electronic devices is not allowed.**

In this exam:  $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ .

1. [6 pt]

Let the sequence of numbers  $a_1, a_2, a_3, a_4, \dots$  be given by:

$$a_1 = 2, a_2 = 6, a_3 = 15, \text{ and for } n \geq 4: \quad a_n = a_{n-1} + 2a_{n-2} + 4a_{n-3}.$$

Prove with mathematical induction that for all  $n \in \mathbb{Z}^+$ ,  $a_n \leq \left[\frac{5}{2}\right]^n$ .

2. [6 pt]

Let  $\mathcal{U}$  be a nonempty universe and let  $f : \mathcal{P}(\mathcal{U}) \times \mathcal{P}(\mathcal{U}) \rightarrow \mathcal{P}(\mathcal{U})$  be the operation on  $\mathcal{P}(\mathcal{U})$  given by

$$f(A, B) = \overline{A \cup B}.$$

Examine if  $f$  is commutative, associative and if  $f$  has an identity.

3. [6 pt]

Let  $A$  be a finite set and let  $R$  be a relation on  $A$ . Let  $M$  be the relation matrix for  $R$ .

Prove that:  $R$  is transitive if and only if  $M^2 \leq M$ .

**Total: 18 points**