Take-home Examination Part 1: Modeling and analysis of concurrent systems (MACS), 2018/2019.

To be handed in before Monday October 22, 18.00h.

- This examination should be made individually. Any form of collaboration with others is considered fraud.
- The work should be handed in in the postbox of Rom Langerak, in INF 3047. No electronic submission.
- Indicate address, student number and study.
- Each question is worth 10 points, except questions 5 and 8 which are both worth 20 points. Mark: total divided by 10.

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1. Consider the two automata on top of page 19 of Milner's book; suppose p_2 and q_1 are the only accepting states. Show that the automata accept the same language (by solving the appropriate equations).
2. Consider the processes
$$P_1 \stackrel{def}{=} a.(b.P_1 + \tau.c.P_1) + a.c.P_1$$
and
$$P_2 \stackrel{def}{=} a.(b.P_2 + \tau.c.P_2)$$
Prove that P_1 and P_2 are weakly bisimilar (by proving that there is a bisimulation relation).
3. Specify a bitstack with capacity 3. Draw the transition system.
4. Give the standard form of
$$(\mathbf{new}a(a.Q + b.S))|(\mathbf{new}b(\overline{a}.R + \overline{b}.S))|(\mathbf{new}c(a.Q + c.Q))$$
and prove that it is structurally equivalent.
5. We introduce a new operator *interrupt*, and denote A *can be disabled by* B by A [> B. The meaning of A [> B is that an action from A can happen, resulting in A' [> B, or an action from B can happen, resulting in B' and thus disabling A.
   (a) Give transition rules for [>.
   (b) Draw the transition system for
   $$a.b.c.0 \text{ [> } d.e.0$$
   and give a derivation for action a followed by action d.
   (c) Show that in the presence of [> weak bisimulation is no longer a congruence (hint: find processes B_1 and B_2 with $B_1 \approx B_2$ but for some B, B [> $B_1 \not\approx B$ [> B_2).
6. Prove using only Theorem 6.15 from Milner's book:
   (a) $a.P \approx a.P + a.P$
   (b) $a.P + \tau.(b.Q + c.R + d.S) \approx a.P + b.Q + c.R + \tau.(b.Q + c.R + d.S)$

7. Consider the following equation:

$$X \approx \tau.X + a.P$$

Prove that for any process Q and any choice expression M, the process

$$\tau.(a.P + \tau.Q) + M$$

is a solution.

8. An agent A transforms an input into an output:

$$A \stackrel{def}{=} i.A', \quad A' \stackrel{def}{=} o.A$$

An agent A is implemented by a protocol entity P that has to grab a device D after an input, then releases it, and then outputs:

$$P \stackrel{def}{=} i.g.r.o.P, \quad D \stackrel{def}{=} \overline{g}.\overline{r}.D$$

We now look at two agents in parallel.

(a) Draw the labelled transition system of $A|A$.

(b) Show that we can implement $A|A$ by two protocol entities and one device, by proving that

$$A|A \approx \mathbf{new}\, gr(P|P|D)$$

(hint: use proposition 6.9 from Milner's book).