Course : **Discrete Mathematics for Technical Computer Science; Part 1**
Date : November 9, 2018
Time : 08.45–09.45 hrs

Motivate all your answers. The use of electronic devices is not allowed.
A formula sheet is included.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

1. [6 pt]
   Let A be a matrix with m rows and n columns. a_{ij} denotes the real number in the cell in row i and column j of A. Give quantified expressions for the following statements.

   (a) All diagonal elements of A are equal to 0.

   (b) The largest value in the first row of A is equal to 7.

2. [6 pt]

(a) Prove the validity of the following argument using the "Laws of Logic" and the "Rules of Inference".

$$\begin{array}{l} q \\ \neg q \vee p \\ (p \vee r) \rightarrow s \\ \hline \therefore (s \wedge r) \vee (s \wedge p) \end{array}$$

(b) Give a counterexample to show that the following argument is invalid.

$$\begin{array}{l} \forall x\ [p(x) \vee q(x)] \\ \hline \therefore \forall x\ p(x) \vee \forall x\ q(x) \end{array}$$

3. [6 pt]
   Let A, B and C be sets in a universe $\mathcal{U}$. Give a proof or a counterexample for the following implication:

$$A \Delta C = B \Delta C \Longrightarrow \quad A = B.$$

Total: 18 points

Course : **Discrete Mathematics for Technical Computer Science; Part 2**
Date : November 9, 2018
Time : 10.00–11.00 hrs

Motivate all your answers. The use of electronic devices is not allowed.
A formula sheet is included.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

4. [6 pt]
Prove with mathematical induction that for all $n \in \mathbb{N}$, $n \geq 66$, n can be written as $n = 7s + 12t$, for some $s, t \in \mathbb{N}$.

5. [6 pt]
Let f be the closed binary operation on $\mathcal{P}(\mathbb{N})$ given by

$$f(A, B) = A \cup B \qquad (A, B \subseteq \mathbb{N})$$

(a) Determine if f is one-to-one; if f is onto; and if f is invertible.

(b) Determine $f^{-1}(\{1\})$.

6. [6 pt]
Let $A \subseteq \mathbb{R}$ and let R be the relation on A^2 given by:

$$(a, b)R(c, d) \quad \text{if and only if } (a + b \leq c + d \text{ and } a - b \leq c - d).$$

(a) Show that (A^2, R) is a poset.

(b) Take $A = \{0, 1, 2\}$. Construct a Hasse diagram for (A^2, R).

Total: 18 points