201300180 Data & Information – Test 2 extra question 3b – Solution

1) First we determine $\mathcal{F}^+$:
   from $C \longrightarrow A$ and $A \longrightarrow E$ we find $C \longrightarrow E$,
   from $BD \longrightarrow C$, $C \longrightarrow A$ and $C \longrightarrow E$ we find $BD \longrightarrow CAE$,
   yielding

   $$\mathcal{F}^+ = \{ A \longrightarrow E,\ BD \longrightarrow CAE,\ C \longrightarrow ADE \}$$

   Candidate keys are *BD* and *BC*. The first one is obvious (from $BD \longrightarrow CAE$).
   But from A*BCDE* we can eliminate A*DE* (using $C \longrightarrow ADE$), leaving *BC* second as candidate key.

   From the functional dependencies in $\mathcal{F}$, $A \longrightarrow E$ *and* $C \longrightarrow AD$ violate the BCNF condition.
   $BD \longrightarrow C$ satisfies the condition, its left-hand side is a superkey.

2) Start with functional dependency $A \longrightarrow E$.

   $A^+ = AE$. Splitting on *A* we get

   - $R_1(A,E)$, with $\mathcal{F}_1 = \{ A \longrightarrow E \}$ (candidate key *A*)
   - $R_2(A,B,C,D)$, with $\mathcal{F}_2 = \{ BD \longrightarrow CA,\ C \longrightarrow AD \}$ (candidate keys *BD, BC*)

   Clearly, R_1 is in BCNF. For R_2 we have one violating functional dependency: $C \longrightarrow AD$.
   $C^+ = CAD$. Splitting R_2 on *C* we get

   - $R_{21}(C,A,D)$, with $\mathcal{F}_{21} = \{ C \longrightarrow AD \}$ (candidate key *C*)
   - $R_{22}(B,C)$, with $\mathcal{F}_{22} = \{ \}$ (candidate key *BC*)
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Alternatively, start with functional dependency $C \longrightarrow AD$.

   $C^+ = CADE$. Splitting on *C* we get

   - $R_1(C,A,D,E)$, with $\mathcal{F}_1 = \{ A \longrightarrow E,\ C \longrightarrow ADE \}$ (candidate key *C*)
   - $R_2(B,C)$, with $\mathcal{F}_2 = \{ \}$ (candidate key *BC*)

   Clearly, R_2 is in BCNF. For R_1 we have one violating functional dependency: $A \longrightarrow E$.
   $A^+ = AE$. Splitting R_1 on *A* we get

   - $R_{11}(A,E)$, with $\mathcal{F}_{11} = \{ A \longrightarrow E \}$ (candidate key *A*)
   - $R_{12}(A,C,D)$, with $\mathcal{F}_{12} = \{ C \longrightarrow AD \}$ (candidate key *C*)

3) From the original functional dependencies, $BD \longrightarrow C$ was lost in the first (resp. second) step.
   The other FDs in $\mathcal{F}$ still exist in $\mathcal{F}_1 \cup \mathcal{F}_{21} \cup \mathcal{F}_{22}$ (resp. $\mathcal{F}_{11} \cup \mathcal{F}_{12} \cup \mathcal{F}_2$).