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Exam Part 2, Web Science 201500025 (Games, Auctions, Voting)

Friday, January 15, 2016, 8.45-11.45

- Use of calculators, mobile phones, etc. is not allowed!
- This exam consists of five problems. Please start a new page for every problem.
- You have 3 h time.
- Please write name and student ID on your solutions.
- Total number of points: 36+4 = 40. Distribution of points:

1a: 2	2a: 3	3a: 3	4a: 2	5a: 3
1b: 2	2b: 3	3b: 3	4b: 2	5b: 3
1c: 2		3c: 2	4c: 1	5c: 5

Question 1

Decide for each of the following statements whether it is true or false. Give a short argument to justify your answer (one or two sentences, or a counterexample). *(2 points per statement)*

(a) For every two-player game, the following holds: If the game possesses a pure Nash equilibrium, then at least one of the players has a dominating strategy.

(b) For every two-player game, the following holds: If both players have strictly dominating strategies, then there is exactly one pure Nash equilibrium.

(c) In every *first*-price sealed-bid auction, the following holds: If all bidders bid truthfully, then all bidders have a revenue of 0.

Question 2

Consider the game specified by the following payoff matrix and answer the following questions.

opponent

		ℓ	m	r
	T	6/4	0/5	4/4
you	M	7/3	1/1	5/2
	B	9/2	3/2	2/1

(a) *(3 points)* Which strategies are strictly dominated by which strategies? Write down the resulting game with the strictly dominated strategies removed.

Does the resulting reduced game have strictly dominated strategies? If yes, by which strategies are they dominated? Iterate the process of removing strictly dominated strategies until no strictly dominated strategy remains.

(b) *(3 points)* Does the resulting game have pure Nash equilibria? If yes, list all pure Nash equilibria. Does the game have mixed Nash equilibria? If yes, compute and describe them.

Question 3

Consider the following example of a sponsored search auction with advertisers X, Y, and Z and three slots.

		10	5	0	clickthrough rate
		slot 1	slot 2	slot 3	
8	X	80	40	0	
6	Y	60	30	0	
4	Z	40	20	0	
value					

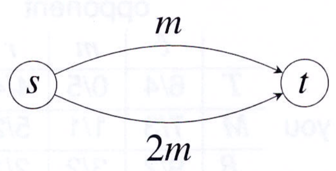
(a) *(3 points)* Compute market-clearing prices and a corresponding assignment of advertisers to slots together with their payoffs for this matching market by raising prices according to the price-raising procedure.

(b) *(3 points)* Run the VCG mechanism to compute an assignment of slots to advertisers and corresponding prices. You only have to compute the relevant personalized prices.
Draw the preferred-seller graph for the corresponding prices (with the price paid as the price for everybody).

(c) *(2 points)* Run a generalized second-price auction for this sponsored search auction. Assume that all three advertisers bid truthfully their values. What are the advertisers' payoffs?

Question 4

Consider the following simple road network. There are 99 cars that want to go from s to t. The travel time on the upper road is always m minutes if there are m cars on that road. The travel time on the lower road is $2m$ minutes if there are m cars on that road.



(a) *(2 points)* Compute the social optimum for this road network.

(b) *(2 points)* Compute a Nash equilibrium for this road network.

(c) *(1 point)* What can you conclude for the price of anarchy for this network routing game?

Question 5

(a) *(3 points)* Consider a voting rule on $m \geq 3$ alternatives in which voters with complete and transitive preferences distribute the scores 5, 3 and 1 over their three most favoured alternatives.

Prove or give a counterexample: If there is a Condorcet winner, then this alternative wins.

(b) *(3 points)* Suppose that you receive the assignment to design a voting rule F that fulfils two requirements: it should be unanimous, meaning that if $X \succ_i Y$ for all voters i in a preference profile P, then $X \succ Y$ in $F(P)$, and it should also be anonymous, meaning that $F(\succ_1, \ldots, \succ_n) = F(\succ_{\pi(1)}, \ldots, \succ_{\pi(n)})$ for any permutation π of the voters. We assume for simplicity that the number of voters is odd.

Give a voting rule that fulfils both requirements. Also give at least one major weakness of *any* voting rule that fulfils these two requirements.

(c) *(5 points)* Consider the following experiment. There are two decks, one with 10 red cards, and one with 1 red and 9 black cards. Let us call the first deck red, and the second black. Any of the two decks is chosen with probability $\frac{1}{2}$. Once that has been done, there are 3 players who get to see, each individually, one random card of the deck. In order to decide if the deck is actually red or black, the players must vote red or black *by majority*. Assume players are non-strategic (i.e., they follow their private signals).

(i) What is the probability that a player sees a red card?

(ii) What is the probability that the players decide red? (Hint: the result is $> \frac{1}{2}$.)

(iii) Suppose the players have decided red. What is the conditional probability that the deck is indeed red?

(iv) Briefly discuss: When a player sees a red card, and is interested in the correct outcome, should he actually be non-strategic and follow the signal?