

MOD7:ADS

1

10 pt

Consider the following algorithm (with * for multiplication, // for integer division (eg. $7//2 = 3$), and **2 for square):

```
def func(n):  
    if n==0:  
        return 1  
    else:  
        if n<4:  
            return n  
        else:  
            return 2*func(n//4) + 6 + func(n//4)**2
```

1. Give a recursive expression for the time complexity of this algorithm, expressed in the number of arithmetical operations.
2. What is the complexity class of this algorithm?

2a

5 pt

Suppose in a heap you update an arbitrary element (say with index i). Describe (in words or in pseudocode) an algorithm that repairs (if necessary) the heap property.

2b

5 pt

Given a binary search tree with positive keys, and a key k that does not occur in the tree. Give a function that yields: the biggest key in the tree, smaller than k (or zero if there is no such key). Hint: traverse the tree as if you want to insert k , and keep track of what you encounter.

3

10 pt

Suppose you want to put songs on a cd. Suppose you can choose from n songs, where song i takes t_i minutes. You want to fill the cd as much as possible, which means that you want to put as much minutes of music on it as possible. Assume a cd may contain at most 80 minutes of music.

1. suppose $C(i, k)$ indicates the minimal remainder (so the amount of unused minutes) if still k minutes need to be filled with songs chosen from the set $\{1, \dots, i\}$. Explain that

$$C(i, k) = \min\{C(i-1, k), C(i-1, k-t_i)\}$$

2. Give a polynomial algorithm, based on dynamic programming, that calculates the maximal amount of minutes you can put on the cd.