

Reference : AM2017/DMMP/010/ha

Course : **Discrete Mathematics for Computer Science**

Date : October 27, 2017

Time : 08.45–09.45 hrs

Motivate all your answers. The use of electronic devices is not allowed.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \dots\}$.

1. [6 pt]

Let the sequence of integers a_0, a_1, a_2, \dots be given by:

$$a_0 = 1, a_1 = 2, a_2 = 7, \text{ and for } n \geq 3: a_n = 2a_{n-1} + a_{n-2} + 2a_{n-3}.$$

Prove with mathematical induction that for all $n \in \mathbb{N}$, $a_n \leq \left[\frac{8}{3}\right]^n$.

2. [6 pt]

Let A, B and C be sets and let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions such that f is onto and $g \circ f$ is one-to-one. Prove that g is one-to-one.

3. [6 pt]

Let $A = \{(x, y) \mid x, y \in \{0, 1, 2\}\}$ and let R be the relation on A given by:

$$(a, b)R(c, d) \quad \text{if and only if} \quad a + d = b + c.$$

Show that R is an equivalence relation on A and determine the partition of A induced by R .

Total: 18 points