Tjeerdsma's Tjeat-sheet

LINEAIR EQUATIONS
A lineair equation is consistent if and only if the rightmost column is *not* a pivot column. (Existence and Uniqueness Theorem).

SPAN
Span {$\mathbf{v}_1$, ..., $\mathbf{v}_p$} is the collection of all vectors that can be written in the form $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + ... + c_p\mathbf{v}_p$ with c_1,..., c_p scalars.

COEFFICIENT MATRIX THEOREM
All following rules are equivalent:
- For each **b** in $\mathbb{R}^m$, the equation A**x** = **b** has a solution.
- Each **b** in $\mathbb{R}^m$ is a linear combination of the columns of A.
- The colums of A span $\mathbb{R}^m$
- A has a pivot position in every row.

TRANSFORMATION MATRIX
For each transformation, there is a unique matrix that behaves exactly like T(x) when multiplying x with A: T(x) = Ax

MAPPINGS: ONTO / ONE-TO-ONE
A mapping $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is said to be **onto** $\mathbb{R}^m$ if each b in $\mathbb{R}^m$ is the image of at least one x in $\mathbb{R}^m$.

A mapping $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is said to be **one-to-one** if each b in $\mathbb{R}^m$ is the image of at most one x in $\mathbb{R}^n$.

T is one-to-one if and only if the equation T(x) = 0 has only the trivial solution.

T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$, if and only if the columns of A span $\mathbb{R}^m$

T is one-to-one if and only if the columns of A are linearly independent.

MULTIPLYING MATRIXES
If A is an m x n matris, and B is an n x p matrix, then the product AB is an m x p matrix:

$$AB = A[b_1 \dots b_p] = [Ab_1 \dots Ab_p]$$

For the product to work, A's columns should match B's rows. A's rows x B's columns form the new matrix. AB ≠ BA

INVERSE OF A MATRIX
Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If ad – bc ≠ 0, then A is invertible and $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

ad-bc = det(A) = determinant.

COLUMN SPACE
The **column space** of a matrix A is the set Col A of all linear combinations of the columns of A.

ECHELON FORM
A rectangular matrix is in echelon form if it has the following three properties:
- All nonzero rows are above any rows of all zeros
- Each leading entry of a row is a column to the right of the leading entry of the row above it.
- All entries in a column below a leading entry are zeros

MULTIPLYING: MATRIX * VECTOR

$$Ax = [a1\ a2 \dots an]\begin{bmatrix} x_1 \\ \dots \\ x_n \end{bmatrix} = x_1a_1 + \cdots + x_na_n$$

PROPERTIES OF PRODUCT Ax
If A is an m x n matrix, u and v are vectors in $\mathbb{R}^n$ and c is a scalar, then:
- A(u+v) = Au + Av;
- A(cu) = c(Au)

DEPENDENCY
An indexed set of vectors is said to be **linearly independent** if the vector equation $x_1v_1 + \dots + x_pv_p = 0$ has only the trivial solution (all x'es 0). The set is said to be **linearly dependent** if there exist other solutions for x that result in 0.

LINEAR INDEPENDENCY IN MATRIX
The columns of a matrix A are linearly independent if and only if the equation Ax = 0 has *only* the trivial solution.

LINEARITY IN TRANSFORMATIONS
A transformation is **lineair** if:
- T(u+v) = T(u) + T(v)
  for all u and v in the domain of T
- T(cu) = cT(u)

TRANSPONSE MATRICES
Transponse matrix of A is A^T: in A^T, A's columns are form the rows for A^T.

$$\begin{matrix} a & b \\ c & d \end{matrix} \text{ results in } \begin{matrix} a & c \\ b & d \end{matrix}$$

FINDING INVERSE OF A MATRIX
add $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ to the end of the to-be-inverted matrix and row-reduce the matrix. When ready, remove $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ from the front of the matrix: Inverted!

SUBSPACES OF $\mathbb{R}^n$
A subspace of $\mathbb{R}^n$is any set H in $\mathbb{R}^n$ that has three properties:
- The zero vector is in H.
- For each u/v in H, u+v is in H
- For each u in H the vector cu is in H.

REDUCED ECHELON FORM
A rectangular matrix is in reduced echelo nform if it has the following properties:
- All properties mentioned at 'Echelon form'
- The leading entry in each row is 1.
- Each leading 1 is the only nonzero entry in it's column.

NONTRIVIAL SOLUTION
The homogeneuous equation Ax = 0 has a nontrivial solution if and only if the equation has at least one free variable.

PARAMETRIC FORM
How to write a solution set in parametric:
1. Make reduced echelon form
2. Express each basic variable in terms of free variables
3. Write a typical solution x as a vector whose entries depend on the free variables
4. Decompose x into a linear combination of vectors using the free variables as parameters

LINEAR DEPENDENCY RULE
A set of two vectors {v1, v2} is only linearly dependent if at least one of the vectors is a multiple of the other.

LINEAR DEPENDENCY RULE #2
If a set contains more vectors than there are entries in each vector, then the set is linearly dependent.

LINEAR DEPENDENCY RULE #3
If a set contains the zero vector, then the set is linearly dependent.

RANK THEOREM
If a matrix A has n columns, then rank A + dim Nul A = n.

BASIS THEOREM
Let H be a p-dimensional subspace of $\mathbb{R}^n$. Any linearly independent set of exactly p elements in h is automatically a basis for H. Also, any set of p elements of H that spans H is automatically a basis for H.

BASIS
A **basis** for a subspace H of $\mathbb{R}^n$ is a linearly independent set in H that spans H.

NULL SPACE
The null space of a matrix A is the set Nul A of all solutions of the homogeneous equation Ax = 0

Tjeerdsma's Tjeat-sheet

THE INVERTIBLE MATRIX THEORM

All following rules are equivalent:

- A is an invertible matrix
- A is row equivalent to the n x n identity matrix
- A has n pivot positions
- The equation Ax = 0 has only the trivial solution
- The columns of A form a linearly independt set
- The linear transformation x -> Ax is one-to-one
- The equation Ax = b has at leat one solution for each b in $\mathbb{R}^n$
- The columns of A span $\mathbb{R}^n$
- The linear transformation x -> Ax maps $\mathbb{R}^n$ onto $\mathbb{R}^n$
- There is an n x n matrix C such that CA or AC = I
- A^t is an invertible matrix.
- The columns of A form a basis of $\mathbb{R}^n$.
- Col A = $\mathbb{R}^n$
- Dim Col A =n
- Rank A = n
- Nul A = {0}
- Dim Nul A = 0
- The number 0 is not an eigenvalue of A.
- The determinant of A is not zero.

PROPERTIES OF DETERMINANTS

Let A and B be n x n matrices.

- A is invertible if and only if det A ≠ 0
- Det AB = (detA)(det B).
- Det A^T = det A
- If A is traingular, then det A is the product of the entries on the main diagonal of A.
- A row replacement operation on A does not change the determinant
- A row interchange changes the sign of the determinant.
- A row scaling also scales the determinant by the same factor.

LEGENDA

HOOFDSTUK 1
HOOFDSTUK 2
HOOFDSTUK 3
HOOFDSTUK 5

PIVOT BASIS FOR COL A

The pivot columns of a matrix A form a basis for the column space of A.

DIMENSION

The **dimension** of a nonzero subspace H, denoted by dim H, is the number of vectors in any basis for H. The dimension of the zero subspace {0} is defined to be zero.

BASIS THEOREM

Let H be a p-dimensional subspace of $\mathbb{R}^n$. Any linearly independent set of exactly p elements in h is automatically a basis for H. Also, any set of p elements of H that spans H is automatically a basis for H.

TRIANGULAR MATRIX

If A is a triangular matrix, then det A is the product of the entries on the main diagonal of A. A matrix is traingular if next to the diagonal, it only contains 0's either on top of the diagonal or under the diagonal.

DETERMINANIT RULES

- If A is an n x n matrix, then det A^T = det A.
- If A and B are n x n matrices, then det AB = (det A)(det B).

EIGENVECTORS INDEPENDENT

If v1, … vr are eigenvectors that corrospond to distinct eigenvalues of an n x n matrix A, then the set v1, … vr is linearly independent.

NO DISTINCT EIGENVALUES

Let A be an n x n matrix whose distinct eigenvalues are $У_1$, …, $У_p$.

- For 1 <= k <= p, the dimension of the eigenspace for $У_k$ is less than or equal to the multiplicity of the eigenvalue $У_k$.
- The matrix A is diagonalizable if and only if the sum of the dimensions of the eigenspaces equals n, and this happens if and only if (i) the characteristic polynomial factors completely into linear factors and (ii) the dimension of the eigenspace for each $У_k$ equals the multiplicity of $У_k$.
- If A is diagonalizable and B_k is a basis for the eigenspace corresponding to $У_k$ for each k, then the total collection of vectors in the sets B_1,…,B_p forms an eigenvector basis for $\mathbb{R}^n$

COORDINATE VECTOR

Each x in the subspace H is created by multiplying the basis B with a **coordinate vector**.

RANK

The **rank** of a matrix A is the dimension of the column space of A.

DETERMINANTS

For n bigger than 2, the determinant of an n x n matrix A = [a_{ij}] is the sum of n terms of the form ±a_{1j} det A_{1j}, with plus and minus signs alternating, where the entries a_{11}, a_{12}, …, a_{1n} are from the first row of A.

INVERTIBLE SQUARE MATRIX

A square matrix A is invertible if and only if det A ≠ 0.

DETERMINANIT RULES

- If A is an n x n matrix, then det A^T = det A.
- If A and B are n x n matrices, then det AB = (det A)(det B).

EIGENVALUES ON TRIAN. MATRIX

The eigenvalues of a triangular matrix are the entries on its main diagonal.

EIGENVECTOR & EIGENVALUE

An **eigenvector** of an n x n matrix A is a nonzero vector x such that Ax = Уx fors ome scalar У. A scalar У is called an **eigenvalue** of A if there is a nontrivial solution x of Ax = Уx.

DIAGONALIZABLE?

An n x n matrix with n distinct eigenvalues is diagonalizable.

DIAGONALIZATION

An n x n matrix A is diagonalizable if and only if A has n linearly independent eigenvectors. In fact, A = PDP^{-1}, with D a diagonal matrix, if and only if the columns of P are n linearly independent eigenvectors of A. In this case, the diagonal entries of D are eigenvalues of A that correspond, respectively, to the eigenvectors in P.