LINEAR ALGEBRA Date : April 01, 2022
Time : 13.45 – 15.45 hrs

First read these instructions carefully:

This test contains 9 exercises. The complete solutions of Exercises 5, 6, 7, 8 and 9 must be accurately written down on a separate sheet, including calculations and argumentation. For the other exercises you are only required to fill in the final answers on the answer sheet at the end of this test. You must hand in this answer sheet as well as your hand written solutions to Exercises 5, 6, 7, 8 and 9.
The use of electronic devices is not allowed.

1. *Fill in your final answer to this exercise on the supplied answer sheet.*
The matrix A is given as follows:

$$A = \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}$$

a) Determine A^0, A^2, and A^T
b) Find all the eigenvalues of $A^{-1} + A$

2. *Fill in your final answer to this exercise on the supplied answer sheet.*
Given the matrix $A \in \mathbb{R}^{5X3}$ and the null space defined as:

$$NullA = \text{Span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

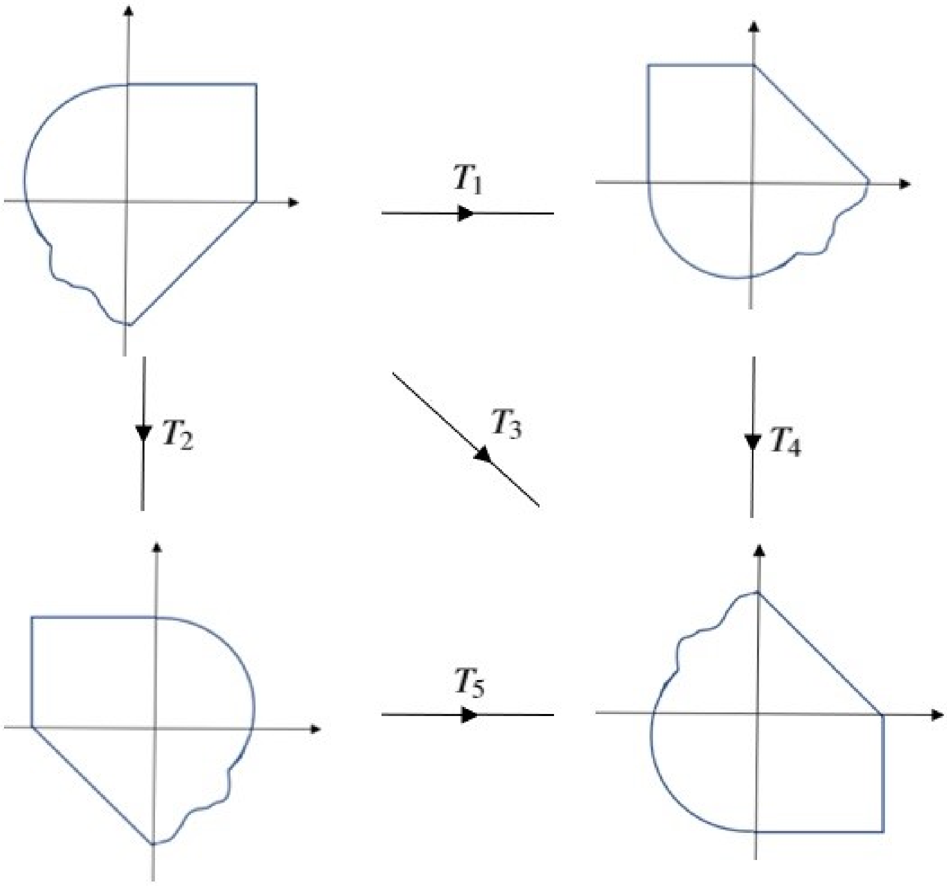
Determine the dimension of Null A.

3. *Fill in your final answer to this exercise on the supplied answer sheet.*
The matrix A is given by:

$$A = \begin{pmatrix} \alpha+5 & 1-\alpha & 0 \\ -\alpha-1 & \alpha+3 & 0 \\ -2\alpha & -2\alpha & 4-2\alpha \end{pmatrix}, \quad \alpha \in \mathbb{R}$$

a) Determine all values of α for which A has eigenvalue 1.
b) For $\alpha = 0$, it is given that one of the eigenvalues of the matrix A is equal to 4. Determine the corresponding eigenspace.

4. *Fill in your final answer to this exercise on the supplied answer sheet.*
We are given three transformations: T_1, T_2, T_3, T_4, and T_5 which have the effect on the figure as shown by the arrows below:



Match each transformation with either of the following matrices from A to H:

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$E = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \quad F = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad G = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}.$$

5. *Use a separate sheet and include clear argumentation and calculation.*
Consider three equations for planes in $\mathbb{R}^3$ are given as follows:

$$\alpha x_1 + \alpha^2 x_2 + 2x_3 = \alpha^2, \quad x_1 + (\alpha - 1)x_2 + \alpha x_3 = 0, \quad x_1 - x_2 + 2\alpha x_3 = \alpha, \quad (\alpha \in \mathbb{R}).$$

Find all the values of α which correspond to the planes intersecting in a line.

6. *Use a separate sheet and include clear argumentation and calculation.*
Let V be a vector space and S_a and S_b be subsets of V.

   a) Determine whether S_a is a subspace of V where:
   $V = \mathbb{R}^2$
   $S_a = \{(x, y) \in \mathbb{R}^2 | 2x - 3y = 6\}$

   b) Determine whether S_b is a subspace of V where:
   $V = \mathbb{R}^n$
   $S_b = \{\mathbf{x} \in \mathbb{R}^n | A\mathbf{x} = 3\mathbf{x}\}$ and A is a particular nxn matrix

7. *Use a separate sheet and include clear argumentation and calculation.*
Consider the subspace $\mathcal{U}$ with basis $\mathcal{B} = \{\mathbf{u}, \mathbf{v}\}$ where $\mathbf{u}=\begin{pmatrix}3\\1\\4\end{pmatrix}$, $\mathbf{v}=\begin{pmatrix}5\\1\\6\end{pmatrix}$, and $\mathbf{w}=\begin{pmatrix}1\\3\\4\end{pmatrix}$.
Show that $\mathbf{w}$ is in $\mathcal{U}$ and determine $[\boldsymbol{w}]_B$.

8. *Use a separate sheet and include clear argumentation and calculation.*
Let $\lambda_1 = 1$, $\lambda_2 = -1$, and $\lambda_3 = 2$ be three eigenvalues of a matrix $A \in \mathbb{R}^{3\times 3}$ with the corresponding eigenspaces E_1, E_{-1}, and E_2 as given below:

$$E_1 = \text{Span}\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\right\}, \quad E_{-1} = \text{Span}\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}\right\}, \quad E_2 = \text{Span}\left\{\begin{pmatrix}1\\0\\1\end{pmatrix}\right\}.$$

Determine the matrix A.

9. *Use a separate sheet and include clear argumentation and calculation.*
Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation such that

$$T\begin{pmatrix}0\\-3\\0\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix}, \quad T\begin{pmatrix}-2\\3\\0\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix}, \quad T\begin{pmatrix}1\\6\\3\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}.$$

   a) Determine the representation matrix of T.

   b) Determine whether T is surjective (onto) and/or injective (one-to-one).

For the exercises the following number of points can be obtained:

Exercise	1a.	3 pts	Exercise	1b.	2 pts	Exercise	2.	2 pts	Exercise	3a.	1 pts
Exercise	3b.	2 pt	Exercise	4.	5 pts	Exercise	5.	3 pts	Exercise	6a.	2 pts
Exercise	6b.	3 pts	Exercise	7.	3 pts	Exercise	8.	4 pts	Exercise	9a.	3 pts
Exercise	9b.	3 pts									

The grade is determined by dividing the total number of points by 4 and adding 1.

Answer sheet

Legibly fill in your answers in the corresponding boxes

Linear Algebra April 01, 2022

Name: ....

Programme: ....

1. a)

   b)

2.

3. a)

   b)

4.