Kenmerk : TW2016/DWMP/010/ha

Course : **Discrete Mathematics for Computer Science**
Date : October 28, 2016
Time : 08.45–09.45 hrs

Motivate all your answers. The use of electronic devices is not allowed.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

1. [6 pt]
   Let the sequence of numbers $a_0, a_1, a_2, a_3, \ldots$ be given by:

   $a_0 = 1$, $a_1 = 1$, $a_2 = 1$, and for $n \geq 3$: $\quad a_n = a_{n-1} + a_{n-3}$.

   Prove with mathematical induction that for all $n \in \mathbb{N}$:

   $$a_{n+2} \geq \left(\sqrt{2}\right)^n.$$

2. Let A, B and C be sets and let $f : A \to B, \quad g : B \to C$ and $h : B \to C$ be functions.

   (a) [4 pt] Show that if f is onto, then

   $$g \circ f = h \circ f \quad \Rightarrow \quad g = h.$$

   (b) [2 pt] Give an example that shows that the implication in part (a) does not necessarily hold is case f is not onto.

3. Let $A = \{1, 2, 3, 4, 5\}$ and let R be the relation on A given by:

   $$xRy \quad \text{if and only if} \quad x^2 - y^2 \quad \text{is divisible by } 8.$$

   (a) [4 pt] Show that R is an equivalence relation on A.

   (b) [2 pt] Determine the partition of A induced by R.

Total: 18 points