Discrete Mathematics for Computer Science

Sample Test, Part 1

Duration: 60 min.
Motivate all your answers.
The use of electronic devices is not allowed.
A formula sheet is included.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

1. $p \in \mathbb{Z}^+$ is called a *prime* if p has exactly two *different* divisors: 1 and p (so 1 is not a prime). Furthermore, $m \in \mathbb{Z}^+$ is called a *perfect square* if m can be written as $m = k^2$ for some $k \in \mathbb{Z}^+$.
   Let $A \subseteq \mathbb{Z}^+$. Give quantified expressions for the following statements. In part (b) you may use the notation $d|n$ for "d is a divisor of n".

   (a) [3 pt] At least one of the elements of A is a perfect square.

   (b) [3 pt] A does not contain any prime.

2. [6 pt]
   Prove the validity of the following argument using the "Laws of Logic" and the "Rules of Inference".

$$\begin{array}{c} (\neg p \vee q) \rightarrow r \\ s \vee \neg q \\ p \rightarrow t \\ (\neg p \wedge r) \rightarrow \neg s \\ \hline \therefore \neg t \rightarrow \neg q \end{array}$$

3. Let A and B be sets in a universe $\mathcal{U}$.

   (a) [3 pt] Prove that: $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

   (b) [3 pt] Prove or disprove (using a counterexample) that $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.

Total: 18 points