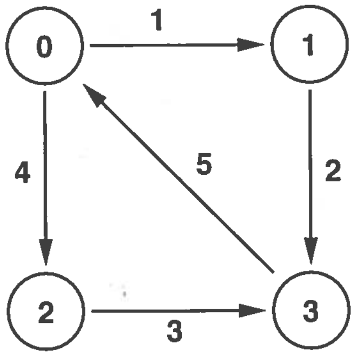
2. Markov chains **(10 points)**

Consider the following CTMC:



(a) Compute the steady-state probabilities of this CTMC using a method of your choice (3 points)

(b) What is the rate at which state 3 is entered in steady-state? (1 point)

(c) Give the transition probability matrix P of the embedded DTMC corresponding to this CTMC. (2 points)

(d) Is this DTMC irreducible? Justify your answer informally. (1 point)

(e) Is this DTMC aperiodic? Justify your answer informally. (1 point)

(f) Is there a way to determine the fraction of time the DTMC spends in each state in the long run? If yes, calculate these fractions. (2 points)

3. Queues (17 points)

Consider two different job classes: class 1 jobs arrive according to a Poisson process with rate $\lambda_1 = 8$ per second and require an exponentially distributed service time which on average takes $E[S_1] = 1/40 = 0.025$ seconds. Jobs in class 2 arrive with negative exponentially distributed interarrival times (parameter $\lambda_2 = 2$ per second) and also have exponential service with $E[S_2] = 1/5 = 0.2$ seconds.

In a first model, jobs are served by two independent M|M|1 queues, one for each job class. The scheduling discipline is FCFS.

(a) Compute the expected waiting times $E[W_1^M]$ and $E[W_2^M]$ for both queues. (1 point)

(b) What is the average waiting time (2 points) $E[W_A]$ for any job (independent of its class)? (1 point)

(c) Indicate the *response* time distribution function $F_{R_1^M}(t)$ for the first queueing station. (1 point)

In a second model, the two job classes share one queueing station.

(d) Indicate

(1) overall arrival rate λ, (1 point)

(2) expected service time $E[S]$ and (1 point)

(3) second moment of the expected service time $E[S^2]$ (1 point)

of an M|G|1-FCFS queueing station that serves both classes.

(e) Compute the expected waiting time $E[W_B]$ for jobs in this M|G|1 queueing station. (1 point)

(f) Now change the scheduling discipline of this M|G|1 queueing station to processor sharing (PS). Compute the expected waiting time $E[W_C]$ for this case. (1 point)

As a third model, both classes are served by an M|G|1 queueing station, but class 1 jobs have priority over class 2 jobs. The queuing station employs the *preemptive resume (PRS)* strategy.

(g) Compute the expected waiting times $E[W_1^P]$ and $E[W_2^P]$ for this scenario. (2 points)

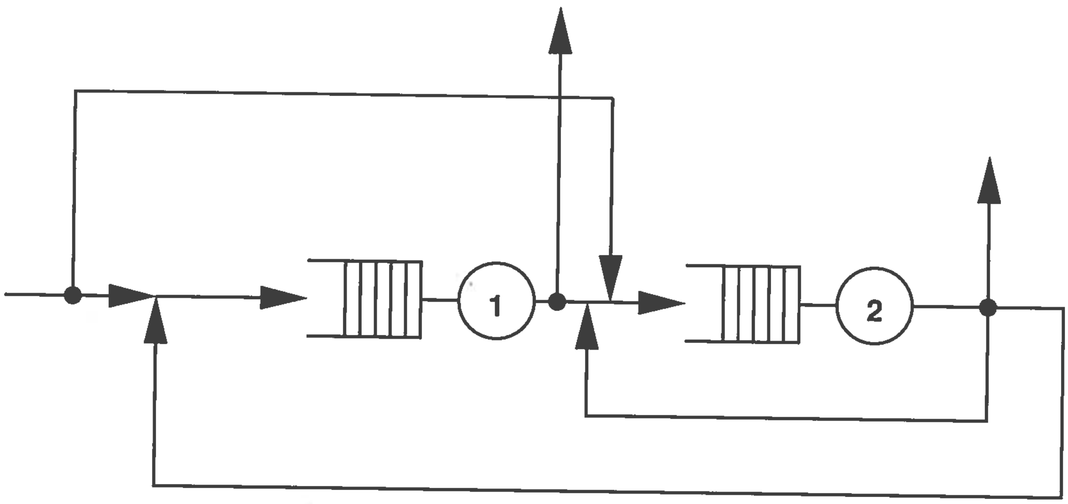
(h) What is the average expected waiting time $E[W_D]$ for a random job in this priority queue?(1 point)

(i) Compare and discuss the values of $E[W_A]$, $E[W_B]$, $E[W_C]$ and $E[W_D]$ as computed in exercises (b), (e), (f) and (h). (2 points)

(j) With the given service time distribution, the M|G|1 queueing station with preemptive priority scheduling can be represented by a CTMC with infinite state space. Draw the state-transition diagram of the M|G|1 with preemptive priority scheduling. A state is hereby a pair (i, j), where i the number of class 1 jobs and j the number of class 2 jobs present in the system. (points) (2 points)

5. Queueing networks (12 points)

Consider the following Jackson network.



$$\lambda_0 = \frac{6}{5} \qquad r_{0,1} = \frac{1}{6}, r_{0,2} = \frac{5}{6}$$

$$\mu_1 = 1 \qquad r_{1,0} = \frac{2}{3}, r_{1,1} = 0, r_{1,2} = \frac{1}{3}$$

$$\mu_2 = 2 \qquad r_{2,0} = \frac{1}{3}, r_{2,1} = \frac{1}{3}, r_{2,2} = \frac{1}{3}$$

(a) (i) State the traffic equations of this Jackson network,
(ii) solve the traffic equations, and
(iii) show that the network is stable.

(3 points)

(b) Compute $E[N_1]$ and $E[N_2]$. (2 points)

(c) Compute the expected response time for jobs entering the system. (2 points)

Now assume that the network is closed, that is, the number of customers K present in the network is constant. Each customer leaving the network towards the environment is directly rerouted into the network.

(d) Set $V_1 = 1$ and determine the visit count V_2. (1 point)

(e) Compute the service demands D_1 and D_2. (1 point)

(f) Which of the two stations is the bottleneck? (1 point)

(g) Using the convolution algorithm, compute the normalising constant $G(M, K)$ for $K = 1, \ldots, 5$. (2 points)