Course : **Mathematics B2 (Newton)**

Date : November 28, 2014

Solutions

1. f continuous at $x = 1$ means $\lim_{x\to 1} f(x) = f(1)$. [1 pt]

   $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^2 + ax) = 1 + a$ [$\frac{1}{2}$ pt]

   $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \dfrac{x^2-1}{x-1} = \lim_{x\to 1^+} \dfrac{(x-1)(x+1)}{x-1} = \lim_{x\to 1^+} (x+1) = 2$ [1 pt]

   Therefore, $f(1) = 1 + a = 2$, so $a = 1$. [$\frac{1}{2}$ pt]

2. Plugging in $t = 0$ gives indeterminate $\frac{0}{0}$, so we use l'Hôpital's rule. [1 pt]

   $\lim_{t\to 0} \dfrac{\ln(2t^2+1)}{t^2} = \lim_{t\to 0} \dfrac{\frac{4t}{2t^2+1}}{2t}$ [1 pt]

   $\lim_{t\to 0} \dfrac{\frac{4t}{2t^2+1}}{2t} = \lim_{t\to 0} \dfrac{2}{2t^2+1} = 2$ [1 pt]

3. For extrema, we investigate critical and endpoints, so $x = 0$, $x = 4$ and all x for which $f'(x) = 0$ or $f'(x)$ does not exist. [1 pt]

   $f'(x) = 0 \Leftrightarrow \dfrac{1}{2\sqrt{x}}(3-x) - \sqrt{x} = 0 \Leftrightarrow 3 - x = 2x \Leftrightarrow x = 1$ [1 pt]

   Extrema candidates are $f(0) = 0$, $f(1) = 2$ and $f(4) = -2$. [1 pt]

   The function must have absolute extrema, so we conclude that $f(1) = 2$ is the absolute maximum and $f(4) = -2$ is the absolute minimum. [1 pt]

4. Polar coordinates: $x = r\cos\theta$ and $y = r\sin\theta$. [1 pt]

   $\dfrac{x^3+y^3}{x^2+y^2} = \dfrac{r^3\cos^3\theta + r^3\sin^3\theta}{r^2} = r\cos^3\theta + r\sin^3\theta$ [1 pt]

   $\lim_{r\to 0^+} (r\cos^3\theta + r\sin^3\theta) = 0$ [1 pt]

   It follows that $\lim_{(x,y)\to(0,0)} \dfrac{x^3+y^3}{x^2+y^2} = 0$. [1 pt]

5. $\dfrac{\partial z}{\partial x} = \sin(x+y) + x\cos(x+y)$ [1 pt]

   $\dfrac{\partial z}{\partial y} = x\cos(x+y)$ [1 pt]

   Equation tangent plane: $z - z_0 = (x - x_0)\left.\dfrac{\partial z}{\partial x}\right|_{(x_0,y_0)} + (y - y_0)\left.\dfrac{\partial z}{\partial y}\right|_{(x_0,y_0)}$ [1 pt]

   Take $x_0 = -1$, $y_0 = 1$ and $z_0 = 0$, we have $z = -x - y$. [1 pt]