Kenmerk: EWI2014/dmmp/026/BM

Exam Limits to Computing (201300042)

Thursday, October 30, 2014, 8:45 – 11:45

- Use of calculators, mobile phones, etc. is not allowed!
- This exam consists of four problems. Please start a new page for every problem.
- Total number of points: 45 + 5 = 50. In addition, Exercise 1d gives 5 bonus points. The distribution of points is according to the following table.

1a: 3	2: 8	3a: 4	4a: 2
1b: 5		3b: 9	4b: 2
1c: 6			4c: 2
1d: 6*			4d: 2
			4e: 2

1. Decidability and Recursive Enumerability

Consider the following problem:

$$\text{NEVERACCEPT} = \{R(M) \mid \text{for all strings } x \in \{0,1\}^\star, M \text{ does not halt on input } x \text{ or } M \text{ rejects } x\}.$$

(a) *(3 points)* Is NEVERACCEPT decidable or undecidable? Prove your answer.

(b) *(5 points)* Is NEVERACCEPT recursively enumerable? Is $\overline{\text{NEVERACCEPT}}$ recursively enumerable? Prove your answer.

Now we consider the following problem:

$$\text{ALWAYSHALT} = \{R(M) \mid R(M) \text{ halts on every input } x \in \{0,1\}^\star\}.$$

(c) *(6 points)* Prove that $\overline{\text{ALWAYSHALT}}$ is not recursively enumerable.

(d*) *(6 bonus points)* Prove that ALWAYSHALT is not recursively enumerable.

2. NP-Completeness

Let $G = (V, E)$ be an undirected graph, and let $U \subseteq V$ be a subset of the vertices.

- We call U a *clique* if $\{u, v\} \in E$ for all $u, v \in U$.
- We call U an *independent set* if $\{u, v\} \notin E$ for all $u, v \in U$.

We consider the following two problems:

$$\textsc{Clique} = \big\{(G, k) \mid G \text{ contains a clique of size } k\big\} \text{ and}$$

$$\textsc{IndependentSet} = \big\{(G, k) \mid G \text{ contains an independent set of size } k\big\}.$$

(8 points) Prove that IndependentSet is $\mathcal{NP}$-complete. (Note that we have proved in the lecture that Clique is $\mathcal{NP}$-complete.)

3. Equality of Complexity Classes

(a) *(4 points)* The class $\mathcal{PSPACE}$ is known from the lecture:

$$\mathcal{PSPACE} = \bigcup_{c>1} \text{DSpace}(n^c).$$

The class $\mathcal{NPSPACE}$ is less famous and defined as follows:

$$\mathcal{NPSPACE} = \bigcup_{c>1} \text{NSpace}(n^c).$$

Prove that $\mathcal{PSPACE} = \mathcal{NPSPACE}$.

(b) *(9 points)* Prove that $\mathcal{NP} = \mathcal{EXP}$ implies $\mathcal{NEXP} = \mathcal{EEXP}$, where

$$\mathcal{EXP} = \bigcup_{c>0} \text{DTime}\left(2^{n^c}\right),$$

$$\mathcal{NEXP} = \bigcup_{c>0} \text{NTime}\left(2^{n^c}\right), \text{ and}$$

$$\mathcal{EEXP} = \bigcup_{c>0} \text{DTime}\left(2^{2^{n^c}}\right),$$

4. Questions

Are the following statements true or false? Give a short justification of your answer.

(a) *(2 points)* For every time-constructable function t, there exists a language L with the following properties:

- $L \in \mathsf{DTime}(2^{t(n)})$ and
- $L \notin \mathsf{DSpace}(t(n))$.

(b) *(2 points)* co-$\mathcal{NP} \cup \mathcal{NP} \subseteq \mathcal{P}^{\textsc{Clique}}$.

(c) *(2 points)* Let A and B be any two decision problems. If A is not decidable and $A \cap B = \emptyset$, then B is decidable.

(d) *(2 points)* If $\mathcal{NP} \subseteq \mathsf{DSpace}(n^2)$, then $\mathcal{NP} \neq \mathcal{PSPACE}$.

(e) *(2 points)* $\textsc{Palindrome} = \{x \in \{0,1\}^n \mid x = x^{\text{rev}}\} \in \mathcal{NC}^1$.