Mathematics C1 (Cayley)

Date : March 27, 2015
Time : 13.45 – 14.45 hrs

The solutions to the exercises need to be well-structured and clearly formulated.
Moreover, you need to motivate your answer in all cases!
The use of electronic devices is not allowed.

1. The matrix A is given by:

$$A = \begin{bmatrix} 3 & -4 & 3 & 1 \\ -1 & 2 & 1 & 3 \\ 2 & -5 & -5 & -11 \\ 4 & -7 & -1 & -7 \end{bmatrix}.$$

   (a) [3 pt] Show that the columns of A are linearly dependent and write one column of A as a linear combination of the other columns.

   (b) [2 pt] Determine a basis for Null A.

2. [5 pt]
   The matrix A is given by: $A = \begin{bmatrix} -6 & 2 \\ 4 & 1 \end{bmatrix}$.

   Determine if A is diagonalizable, and if so, determine an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

3. $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that first reflects each point $(x_1, x_2) \in \mathbb{R}^2$ through the x_2-axis and projects it onto the line $x_2 = x_1$.

   (a) [3 pt] Determine the representation matrix of T.

   (b) [2 pt] Examine if T is one-to-one and if T is onto.

4. [3 pt]
   Let A and B be invertible $n \times n$-matrices.
   Prove, using determinants, that AB^TA^{-1} is invertible.

Total: 18 points